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BALLISTICS

PART I.

Prepared for the Department of Artillery and Gun Defense

by

ALSTON HAMILTON, Captain, Coast Artillery Corps

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by

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ERRATA.

Page.	Line.	
5	25,	read “trajectory” instead of “trojectory.”
5	32,	read 2250 instead of 2200.
14	17,	read “Z” instead of “z” and “z_0” instead of “Z_0”.
15	27,	insert a comma after “feet”.
28	3,	add “(See pp. 70 to 75).”
31	2,	erase “miles per hour.”
32	8,	read “W_y” instead of “W_z.”
32	9,	read “W_y” instead of “W_z.”
52	12,	read “charge” instead of “change”.
53	36,	insert a comma after “used”.
54	36,	insert “or the deflection scale, as the case may be” after “seat”.
58	1,	erase decimal point before “89¾%”.
62	28,	read “temperature” instead of “tempature”.
67	4,	read “ f_a” instead “ fa”.
75	27,	read “U S E” instead of “U S &”.
77	1,	read “to” instead of “we may”.
77	10,	read “$C_0 = \frac{2\,w^2}{c\,d^5}$” instead of “$C_0 = \frac{2\,w^2}{c\,\gamma'\,d^5}$”.
79	7,	read “20” instead of “18”.
79	21, 22,	read “range correction” instead of “corrected range”.
80	26,	read “V” instead of “v”.
88	22,	read “adequate” instead of “ adequatc”.

CHAPTER I.

Exterior Ballistics concerns itself with the behavior of projectiles in their motion through the air.

In order to avoid unnecessary complexity the following hypotheses are made as a basis :

1. That the projectile travels through still air of uniform density.
2. That the resistance of the air is tangential and, at any instant, is directed along the longitudinal axis of the projectile.
3. That the resistance of the air for a given projectile is a function of the velocity alone.
4. That the surface of the earth is a plane and without motion.
5. That gravity is constant in intensity and direction.

The errors incident to these hypotheses are corrected, with some exceptions, by the use of mean values and where practicable by formulas involving known elements, or by actual determination of the range- or elevation-corrections due. In the excepted cases, the exact solution of the problem would demand great complexity of equipment in the position-finding service, due to elements varying with the locality, circumstances and direction of fire; and the corrections are not large enough to justify this, or to produce a material effect. Hence they are ignored, or, where it is advisable to do so, they are represented by mean values.

As Part I., is necessarily short, it will be impracticable to demonstrate the principal formulas on account of the amount of analysis involved, though wherever it is practicable to demonstrate a formula with simplicity, this will be done.

In order to fix ideas and to give a clear conception ot the limitations of even the best projectile, the unresisted motion of a projectile will be first considered. This is technically known as motion *in vacuo*, and the trajectory is referred to as the *trajectory in vacuo*. It is clear that if we suppose the air gradually thinned and reduced to nothing, a projectile will experience less and less retardation and, finally, none at all, except that due to gravity in the ascending branch, which becomes, in the descendin branch, an acceleration, bringing the projectile back to the level from which it started with the same velocity that it had at at the muzzle.

THE TRAJECTORY IN VACUO.

We shall approach the discussion with the following statement of the problem :

A projectile is fired with a muzzle velocity V, and a quadrant angle of departure φ ; it is subject to no other influence except that of the attraction of gravity, which will be represented by g.

(a) Find at the end of any time, t seconds after the gun is fired, what will be the horizontal range x, the height, y, the the remaining velocity v, and the inclination θ ; and their relation to one another.

(b) Find these elements for the point of fall, and represent them by corresponding capital letters.

(c) Find the coördinates, x_0, y_0, of the summit of the trajectory and the time of flight t_0, to this point.

In order to solve this problem we need only algebra and a very elementary knowledge of trigonometry.

(a) Let us suppose first that gravity does not act ; then at the end of the time t, the projectile will have gone Vt feet along the line of departure, corresponding to a horizontal distance $x = Vt \cos \varphi$, and a vertical distance $y = Vt \sin \varphi$. Now the the action of gravity will not affect x, since it acts in a vertical direction only. It will, according to the law of falling bodies, cause the projectile to drop from the line of departure, vertically, a distance $\frac{1}{2} gt^2$; and it will reduce the vertical velocity by gt.

Hence we have, *in vacuo*,

$$x = Vt \cos \varphi \quad - \quad - \quad - \quad - \quad - \quad (1)$$

$$y = Vt \sin \varphi - \tfrac{1}{2} gt^2 \quad - \quad - \quad - \quad - \quad (2)$$

$$v \cos \theta = V \cos \varphi \quad - \quad - \quad - \quad - \quad - \quad (3)$$

$$v \sin \theta = V \sin \varphi - gt \quad - \quad - \quad - \quad - \quad (4)$$

From (1) we find $t = \dfrac{x}{V \cos \varphi}$, and substituting this in (2) we have

$$\frac{y}{x} = \tan \varphi - \frac{gx}{2V^2 \cos^2 \varphi} \quad - \quad - \quad - \quad - \quad (5)$$

From (3), (4), and (1) we find, by eliminating v and t

$$\tan \theta = \tan \varphi - \frac{gx}{V^2 \cos^2 \varphi} \quad - \quad - \quad - \quad (6)$$

We may now find x from (1); y from (5); θ from (6) and v from (3), in the order named.

(b) At the point of fall the projectile has again reached the level of the gun, hence $y = 0$.

Then, from (5),

$$X = \frac{V^2 \sin 2\varphi}{g} \quad - \quad - \quad - \quad - \qquad (7)$$

$$Y = 0$$

and from (6) by substituting for X its value from (7)

$$\tan \omega = \tan \varphi - 2 \tan \varphi$$

$$\text{whence } \omega = -\varphi^* \quad - \quad - \quad - \qquad (8)$$

and since $v_\omega \cos \omega = V \cos \varphi,$

we have

$$v_\omega = V \quad - \quad - \quad - \qquad (9)$$

From (1) and (7) we find $T = \dfrac{2V \sin \varphi}{g}$ - - - (10)

(c) The summit is reached when the vertical velocity is zero; then, equation (4) gives us

$$t_0 = \frac{V \sin \varphi}{g} \quad - \quad - \quad - \qquad (11)$$

Hence from (10)

$$t_0 = \tfrac{1}{2} T \quad - \quad - \quad - \quad - \qquad (12)$$

From (2), (11) and (12) we find

$$y_0 = \tfrac{1}{8} g T^2 \quad - \quad - \quad - \quad - \qquad (13)$$

and from (1), (7) and (13)

$$y_0 = \tfrac{1}{4} X \tan \varphi$$

From the above we see that, with no retardation, we have

$$\sin 2\varphi = \frac{gX}{V^2} \text{ or } \frac{\sin 2\varphi}{X} = \frac{g}{V^2}$$

$$T \cos \varphi = \frac{X}{V} \text{ or } V T \cos \varphi - X = 0$$

$$\omega = \varphi \text{ or } \frac{\tan \omega}{\tan \varphi} = 1$$

$$v_\omega \cos \omega = V \cos \varphi \text{ or } v_\omega = V \cos \varphi \sec \omega$$

$$y_0 = \tfrac{1}{4} X \tan \varphi = \tfrac{1}{8} g T^2.$$

In air, forms analogous to these are useful in direct fire.

---

* The angle of fall, ω, is recognized as negative but will be spoken of as positive since we are concerned only with its magnitude or absolute value. The position angle, the value of whose tangent is y/x is designated ε and is regarded as an absolute value in this discussion since this prevents any confusion; and simplicity is secured, as in Coast Artillery work it is always applied in the same direction.

For high angle fire we use by preference the ratios

$$\frac{10\,X}{V^2};\qquad \frac{10\,T}{V};\qquad \frac{v_\omega}{V};\qquad \frac{y_0}{X}$$

and the angle ω.

The reason for this is that in direct five we have a constant velocity and vary the elevation. In high angle fire we have certain elevation limits and muzzle velocities vary with the zones used ; besides, the simplicity of the law of retardation renders it desirable to tabulate these values on pages, one page for each degree of φ; and φ is the range table argument.

THE PRINCIPLE OF THE RIGIDITY OF THE TRAJECTORY.

This "principle" assumes that, if the angle of departure necessary to reach a certain point at a horizontal range, x, from the gun and on the same level, is known, it will only be necessary, in order to reach another point h feet below the former and at the same horizontal range, x, to subtract from the first angle of departure, the angle ε, called the position angle and given by the equation

$$\tan \varepsilon = {}^{h}/_{x}$$

This is a useful conception, and is always assumed, in direct fire. It is indispensable in Case I, and furnishes a simple and satisfactory method for finding quadrant elevations for use with Case II.

The assumption, while sensibly true, is not exact, and the error in elevation thereby introduced will now be found.

(a) Denote by φ_x the angle of departure that will bring the point of fall to a horizontal range x, with a given muzzle velocity V.

(b) Denote by φ the angle of departure that will deliver the projectile at a distance h feeet below its position in (a), and at the same range x.

Equation (5) then becomes, in the two cases, since y is zero for (a) and — h, for (b).

$$\sin 2\varphi_x = \frac{gx}{V^2} \qquad \text{- - - -} \qquad (a)$$

and

$$\tan \varepsilon = -\frac{y}{x} = \frac{gx}{2V^2\cos^2\varphi} - \tan\varphi = \frac{h}{x}$$

or

$$\tan \varepsilon = \tan\varphi \left\{ \frac{gx}{V^2 \sin 2\varphi} - 1 \right\} \qquad \text{- -} \qquad (b)$$

Hence

$$\tan \varepsilon = \tan\varphi \left\{ \frac{\sin 2\varphi_x}{\sin 2\varphi} - 1 \right\}.$$

From this

$$\sin 2\varphi_x - \sin 2\varphi = 2\cos^2\varphi \tan \varepsilon \qquad - \qquad - \qquad (14)$$

Now, assuming the principle of the rigidity of the trajectory the sight angle of departure used is $\varphi' = \varphi + \varepsilon$.

Substituting $\varphi' - \varepsilon$ for φ in equation (14), we will find after reduction

$$\sin 2\varphi_x = \sin 2\varphi' \left\{ 1 + \tan \varphi' \tan \varepsilon \right\}$$

What we assume, then, under the principle of the rigidity of the trajectory is that

$\tan \varphi' \tan \varepsilon$ is zero or negligible.

The amount of error in φ thereby introduced can be readily ascertained from this equation and its range equivalent found from a range-table.

It is to be noted that φ_x is the φ of a range table, and that when we correct for height of site and curvature* we introduce ε assuming that $\varphi_x = \varphi' = \varphi + \varepsilon$.

The assumption that the principle of the rigidity of the trajectory is applicable in direct fire is productive of no error that is worthy of note. Such error as is produced is just about compensated for by the difference in height above sea-level between the gun and the target, corrections for altitude being made for the height of the gun as if the entire trajectory were above that level. This simplifies the correction for altitude also.

The term "rigidity" in this connection refers to the supposed rigid shape of the trajectory and its chord (drawn from the muzzle to the target or point of impact); and the assumption practically involves the hypothesis, that the figure whose outline is composed of the trajectory and its chord, behaves as if it were cut out of card board and rotated up or down with the muzzle of the gun as a center.

EXAMPLES.

1. Given $V = 2200$, $X = 24000$ feet, compute remaining elements of the trajectory for each thousand yards of range, i.e., for each 3000 feet of range. This includes y, t, v and θ.

(b) Compute also x_0 and y_0.

(c) Taking the value of φ which was found, enter the 12' Range Table and take from it the range, maximum ordinate and angle of fall.

(d) Note the effect of air resistance even on this heavy projectile.

($g = 32.16$ foot-seconds per second.)

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* See Appendix VII.

2. From the 12″ Range Table $V = 2250$ f.s. opposite the range 10000 yards, take out φ, T, y_0, v_ω, ω.

Calculate from these the values

$$\frac{V T \cos \varphi}{X}; \quad \frac{V^2 \sin 2\varphi}{gX}; \quad \frac{v_\omega \cos \omega}{V \cos \varphi}; \quad \frac{4y_0}{X \tan \varphi}; \quad \frac{8y_0}{gT^2}.$$

These values are unity for the conditions in vacuo. Note the effect of air resistance.

3. Assuming the curvature of the earth to produce angular depression of an object on the water surface at the rate of 1 minute for every 4000 yards of range; find ε for each 4000 yards of range for a height above sea level of 65 feet.

Apply the depression due to height of site and curvature, at each of the ranges 4000, 8000 and 12000 yards, to the values of the angles of departure given in a 12″ range table for $V = 2250$ and give the resulting quadrant angles of departure.

(b) Find these values of ε from Artillery Notes, No. 29, and compare results.

(c) What principle is assumed in making these corrections?

Answer to (b)

0°19′.6 for 4000 yards.
0°11′.3 for 8000 yards.
0°09′.2 for 12000 yards.

CHAPTER II.

THE BALLISTIC COEFFICIENT; AND MOTION IN AIR.

A projectile in its flight through the atmosphere suffers retardation due to air-resistance. The resultant direction of this resistance at any point of the trajectory is directly opposite to that of the motion of the projectile. In overcoming this resistance the projectile performs work, the amount of which in a given interval is measured by the loss of kinetic energy of the projectile. This work is also measured by the product of the mean resistance (encountered by the projectile in this interval) and its path in feet.

Let $\rho =$ mean resistance in pounds.
$w =$ weight of projectile in pounds.
$d =$ diameter or caliber of projectile in inches.
$r =$ the retardation of this projectile due to ρ.
$v_1 =$ the velocity of the projectile at the beginning of the interval.
$v_2 =$ the velocity of the projectile at the end of the interval.
$g =$ the acceleration due to gravity $= 32.16$ f.s. per sec.
$m =$ the mass of the projectile $= \frac{w}{g}$.
$a =$ the length of path considered.

Then

$$\text{work done} = \frac{mv_1^2}{2} - \frac{mv_2^2}{2} = \rho a$$

whence

$$\rho a = \frac{w}{g}\left(\frac{v_1 + v_2}{2}\right)(v_1 - v_2) \qquad (1)$$

or

$$\rho = \frac{w}{ga}\left(\frac{v_1 + v_2}{2}\right)(v_1 - v_2)$$

The resistance is numerically given by the product of the mass by the retardation; that is

$$\rho = \frac{w}{g} \cdot r$$

If, now, with a view to establishing the relation between the velocity and retardation a great number of shots are fired, and the resistances and retardations computed and tabulated in some form, we may begin to predict what will happen when a certain projectile is fired with a certain velocity in atmosphere of a certain density ; for we would then know the law of decrease of velocity, at least practically.

This has been done in a most systematic manner ; and an explanation of the general character of the method involved is not only interesting but essential to a knowledge of the elements of ballistics.

Shots are fired through screens at carefully measured intervals and as a shot passes through a screen an electric circuit is broken. Elapsed time between the screens of a pair is thus registered or computed from registered data, and the distance divided by the elapsed time gives the mean velocity which is taken as the velocity at a point midway between the two screens. A second pair of screens gives a second velocity, of the projectile at a second point. We thus have v_1, v_2, and a of formula (1).

The screens of a pair are placed at such a distance apart as to make the assumption that the mean velocity is the velocity at the middle point, practically correct ; and, at the same time, far enough apart to make the percentage error in the determination of the elapsed time sufficiently small.

Thus were determined data regarding different projectiles and under different atmospheric and other conditions.

The results were standardized and made accessible for application to *any* conditions by proper modifications.

In order to show how this data could be so standardized, the following discussion would appear necessary :

A projectile of a certain form assumed as standard and moving with a velocity v in an atmosphere of density δ_1, assumed as standard, does work in moving one foot along its path, which is numerically equal to the mean resistance, since the path is unity. The volume of air encountered in moving one foot is proportional to the area of cross-section of the projectile and therefore to $\frac{\pi d^2}{4}$ or to d^2, the square of its caliber. Hence the retardation is also proportional to d^2. The resistance is seen to be dependent on the velocity of the projectile and on the caliber, but not on the weight. The retardation, from the equation

$$r = \frac{g}{w} \rho$$

is seen to vary inversely as the weight. Hence the retardation varies directly as the square of the caliber and inversely as the weight; and it is different for different velocities.

Let us assume that we have found the retardation pertaining to a projectile, as above, in air *not* of standard density. It is desired to find what it *would have been* for air of standard density. The weight of air in a given volume is proportional to its density; hence the resistance varies directly as the density of the air; and so, therefore, does the retardation. Denoting the corresponding retardation in standard atmosphere by r_s, and standard air density by $\delta_{\prime}$, we have

$$r_s : r :: \delta_{\prime} : \delta$$

or*

$$r_s = \frac{\delta_{\prime}}{\delta} \cdot r$$

In this way we might find a set of values for the retardation for different velocities for a given projectile of standard form, of weight, w, and caliber, d.

If then r_1 is the retardation suffered at any particular velocity by a projectile of one pound weight, one inch caliber, of standard shape, and moving in the standard atmosphere, we would have for the retardation for that velocity of *any* projectile of standard form moving in an atmosphere of density δ

$$\delta_{\prime} \cdot \frac{1}{1} \cdot r = \delta \cdot \frac{d^2}{w} \cdot r_1 \qquad \text{or } r_1 = \frac{\delta_{\prime}}{\delta} \cdot \frac{w}{d^2} \cdot r$$

Denoting $\frac{\delta_{\prime}}{\delta} \cdot \frac{w}{d^2}$ by C we have

$$r_1 = Cr$$

Hence by multiplying the actual retardation of an experimental firing by C we standardize it; and if we tabulate the values of r_1, we may obtain the corresponding values of r when the data for C are known.

It is interesting to note the character of C as thus far described.

Let a projectile of standard form, caliber 1 inch, weight 1 pound, and moving in an atmosphere of standard density, $\delta_{\prime}$, be termed the *unit projectile;* then the retardation for any projectile, for a given velocity is inversely proportional to C, since it is given by the ratio of r_1 to C.

C therefore represents the inertia of the projectile under the conditions; or its power to retain its velocity.

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* See Appendix I.

Since, other things being equal, retardations vary inversely as the masses on the application of a given force, it follows that we may conceive of C as the fictitious weight of a projectile of 1 inch caliber, and of standard form, which would traverse the standard atmosphere with the same facility as the actual projectile (of standard form) traverses the actual atmosphere.

C is called the ballistic coefficient of the projectile under the conditions. But the shape of the projectile may not be standard. Suppose that its head is differently shaped and that the projectile encounters greater or less resistance than that due to one otherwise the same but of standard form. The effect is the same as if the area of cross-section were suitably increased or diminished.

If we introduce a factor c such that the value cd^2 would be equal to the square of the caliber of a projectile (of standard form but otherwise the same as the given projectile) which would encounter the same resistance, we have for the *given* projectile

$$C = \frac{\delta_1}{\delta} \cdot \frac{w}{cd^2}$$

This c is called the coefficient of form and is habitually determined by experiment.

Recent experiments have given data and conclusions from which the following formula has been derived

$$c = \frac{2k}{n}\sqrt{\frac{4n-1}{7}}$$

in which n is the radius of the ogive in calibers and k a constant which is ordinarily unity but which allows for peculiarities of conformation not attributable to the radius of the ogive ; k is determined by experiment.

THE ALTITUDE FACTOR.

Thus far we have supposed the projectile to be moving in an atmosphere of uniform density. But we know that, as the projectile rises above the level of the gun, the density of the air diminishes, obeying the barometric law.

If δ_h be the density of an atmosphere, which. being uniform throughout, would give the same range as the actual atmosphere, other conditions being the same, δ_h would be the mean density of the air encountered by the projectile in its flight and should be substituted for δ in the ballistic coefficient ; we would then have instead of $\frac{\delta_1}{\delta}$ the ratio $\frac{\delta_1}{\delta_h}$.

But $\frac{\delta_{\prime}}{\delta}$ is already determined and δ_h is, as yet, unkown; hence we proceed as follows

$$\frac{\delta_{\prime}}{\delta_h} = \frac{\delta_{\prime}}{\delta} \cdot \frac{\delta}{\delta_h},$$

and if we denote $\frac{\delta}{\delta_h}$ by f_a we have

$$\frac{\delta_{\prime}}{\delta_h} = \frac{\delta_{\prime}}{\delta} \cdot f_a$$

The altitude factor increases as the maximum ordinate increases and has for its mean value for direct fire*

$$f_a = 1 + .000020\, y_0$$

or f_a is greater than unity by 2% for every thousand feet of maximum ordinate, for average conditions at the earth's surface.

THE WIND FACTOR.

This is derived from the expression for the retardation and takes into account the increased wind effect as the projectile rises. It has for its expression†

$$f_w = 1 \pm \frac{2W_x T^{\frac{5}{4}}}{X}$$

The positive sign is used for a rear and the negative for a head wind component.

W_x is the range component of the wind in miles per hour; f_w is a factor of the ballistic coefficient which takes into account the range effect of wind; T is the time of flight in seconds; and X the range in feet.

THE REDUCING FACTOR.‡

Heretofore we have considered the resistance of the air to the motion of the projectile, without regard to its length or to the constraint which it suffers in adapting itself to a curved trajectory; or to its oblique presentation to the air.

The following value of a reducing factor has been found to give close accord with experimental firings:

$$\gamma = \frac{d^3}{2w}\left\{1 + \frac{\varphi + \omega}{135}\right\}.$$

---

* See Appendix I.

† See Appendix II.

‡ See Appendix VI.

This factor is thus dependent on φ and ω which must be found from a trial C taken as $\frac{w}{cd^2}$; then, using the φ and ω, together with f_a (computed by this means) find a new C, which use for a new set of values of y_0, φ and ω, for a more accurate C, etc. until the values of C cease to change sensibly. C is then known for the range and the corresponding line of the range table may be computed.

The ballistic coefficient is, then, in its general form,

$$C = \frac{\delta_{\prime}}{\delta} \cdot f_a \cdot f_w \cdot \frac{w}{c\gamma d^2}$$

For range table conditions, we have

$$C = \frac{f_a}{c\gamma} \cdot \frac{w}{d^2}.$$

Exercise:

Given: thermometer 89°, barometer 29.58 in., $w = 1046$ lbs., $d = 12$; $\varphi + \omega = 15°.75$; $y_0 = 923$ feet; $T = 15.11$ sec. $X = 27000$ feet; $n = 2$ (radius of ogive in calibers).

(a) Calculate f_a, using y_0 as given

(b) Calculate f_a, using $y_0 = \frac{1}{8} gT^2$

(c) Find $\delta_{\prime}/\delta$ from table in Artillery Notes, No. 25.

(d) Find f_w for a 10 mile head wind.

(e) Calculate c.

(f) Calculate γ.

CHAPTER III.

THE USE OF TABLE II, ARTILLERY CIRCULAR M.

This table gives the ballistic functions for direct fire, with arguments V (the muzzle velocity) and $Z = \frac{X}{C}$, (the range for unit projectile).

This table is in convenient form, since, in direct fire, the range tables are computed with a given V, data for C, and an assumed range X feet or R yards, (so that $X = 3R$); and with $\frac{X}{C}$ and V we may find from Table II, values giving means of promptly computing the other elements.

The functions of which need will be had in Part I. are, for the point of fall,

$$A = \frac{\sin 2\varphi}{C} \qquad (A)$$

$$\log B' = \log \left(\frac{\tan \omega}{\tan \varphi}\right) \qquad (B)$$

$$u = v_\omega \cos \omega \sec \varphi \qquad (C)$$

$$T' = \frac{T}{C} \cos \varphi \qquad (D)$$

$$\log C' = \log \frac{\sin 2\varphi}{X}. \qquad (E)$$

The functions which pertain to the summit are found in A′ and A″ columns, and these functions will be designated as a_0' and a_0'' in our formulas.

The use of these is as follows:

An important property of the trajectory is shown by Colonel Ingalls to be that

$$A = a_0'$$

Hence, knowing A for the point of fall, it is only necessary to place a_0' equal to this value, find this value in the A′ column and take out (from the A″ column) a_0'', on the same horizontal line as a_0'; also z_0, if desired, from the Z column.

Then, a_0'' is the symbol for $\frac{y_0}{C \tan \varphi}$, so that $y_0 = a_0'' C \tan \varphi$.

To use these functions, we have

$$\sin 2\varphi = AC, \text{ (whence } \varphi) \tag{I}$$

$$A = a_0', \text{ whence } a_0'' \text{ and } z_0 \tag{II}$$

$$x_0 = Cz_0 \tag{III}$$

$$y_0 = a_0'' C \tan \varphi \tag{IV}$$

$$\tan \omega = B' \tan \varphi, \text{ (whence } \omega) \tag{V}$$

$$v_\omega = \frac{u \cos \varphi}{\cos \omega} = u \cos \varphi \sec \omega \tag{VI}$$

$$T = C\,T' \sec \varphi. \tag{VII}$$

In these tables will be noticed two columns after each of the functions; these columns are headed $\triangle_z$ and $\triangle_v$.

Of these, use $\triangle_z$ alone. It is the difference between the values of the function (A, for instance) corresponding to the values of Z on the same line and on the next lower line.

The use of $\triangle_v$ leads to some inaccuracy and this increment will therefore not be used for interpolating.

Illustrations will now be given of the use of the tables.

(a) Given V = 2200, Z = 3600, find A, Z_0, a_0'', log B', u, T', log C'.

zo

Turn to page 80 of Table II, Artillery Circular M. Note that the V with which the table is identified is the first of the two values mentioned at the top of the page. In this instance we find at the head of the page "Table II. V = 2200 to 2300 f.s."

Ignore the words "to 2300" as they refer to the use of $\triangle_v$. (It is well to remember this to avoid getting on the wrong V page.)

In the Z-column find the number 3600; on the same horizontal line we find functions as follows:

A	log B'	u	T'	log C'
.03335	0.1353	1377	2.082	4.9667

Hence

$a_0' = .0334$. This number, when found in the A' column, is opposite some value of z between 1900 and 2000. Using the $\triangle_z$ column we find $z_0 = 1900 + \frac{9}{22} \times 100 = 1941$; a_0'' is found similarly to be $1028 + \frac{41}{100} \times 58 = 1052$.

(b) Given V = 2200, z = 6085, find the same functions.

Here it will be necessary to interpolate for each function on account of the non-tabular Z, as was done in (a) in finding a_0' from z_0.

The results will be as follows:

$V = 2200$

Z	A	a_0''	log B′	u	T′	log C′
6085	.07310	1964	.2122	1061	4.169	5.0796

(c) Given $V = 2274$; $Z = 6085$, find the functions.

To do this we find these values for $Z = 6085$, for $V = 2200$ and then for 2300, and interpolate between these values to find them for $V = 2274$, as follows:

$z = 6085$

	V	A	a_0''	log B′	u	T′	log C′
	2300	.06675	1968	.2171	1092	3.994	5.0402
	2200	.07310	1964	.2122	1061	4.169	5.0796
△	100	—.00635	+4	+.0049	+31	—0.175	—.0394
△	74	—.00470	+3	+.0036	+23	—0.130	—.0292
add	2200	.07310	1964	.2122	1061	4.169	5.0796
Res.	2274	.06840	1967	.2158	1084	4.039	5.0504

(d) A special and useful case of (c) is where V is half way between two tabular values, for example where it is 2250. In such a case take the half-sum of the functions as determined for the tabular V's. Thus for $V = 2250$.

$z = 6085$

	V	A	a_0''	log B′	u	T′	log C′
	2300	.06675	1968	.2171	1092	3.994	5.0402
	2200	.07310	1964	.2122	1061	4.169	5.0796
Sum	4500	.13985	3932	.4293	2153	8.163	10.1198
Half sum	2250	.06993	1966	.2147	1077	4.082	5.0599

Exercises:

1. (a) Take $V = 2250$, Range 14200 yards = 42600 feet, $C = 7$ (so that $Z = 6085$), and calculate by the use of values found in (d) for $V = 2250$, the values of φ, y_0, ω, v_ω, T; and check φ by computing it from log C′.

(b) Show that $C' = \frac{A}{Z}$, or $\log C' = \log A - \log Z$, remembering that $C' = \frac{\sin 2\varphi}{X}$.

(c) Using A, V, Z, find the value of $\frac{V^2 \sin 2\varphi}{gX}$, (which in vacuo is unity.)

(d) Using V, T′ and Z find the value of $\frac{V\,T \cos \varphi}{X}$, (which equals unity in vacuo.)

(e) Using u and V find $\frac{v_\omega \cos \omega}{V \cos \varphi}$.

(f) Using log B′ show value of $\frac{\tan \omega}{\tan \varphi}$.

2. (a) Given $\varphi = 5° 44' 45''$; $X = 23400$ ft; $V = 2250$ f. s. calculate log C′.

(b) Find from V and log C′, Z, a_0'', log B′, u and T.

(c) Calculate $C = \frac{X}{Z}$ and then y_0, ω, v_ω, and T.

(d) Given $w = 1046$; $d = 12$; $\delta_{/\!/}/\delta = 0.92$; $f_w = 0.98$; find f_a, γ and the coefficient of form, c.

(e) Given n = 2 calibers, find k.

CHAPTER IV.

THE COMPUTATION OF RANGE TABLES.

In the range-table we must assume certain conditions to obtain. These are

1. That the gun is on the same level as the target; that is that the quadrant elevation is also the sight elevation the position angle being zero.
2. That the value of δ_1 / δ is unity.
3. That there is no wind.
4. That the projectile has a certain muzzle velocity V.
5. That the projectile used is of a given calibre, weight and description.

   These are called *normal* conditions.

Any departure from these conditions must be taken account of for the particular battery.

Thus :

1. For a given height of site the range-table elevations, which on the principle of the rigidity of the trajectory, are sight elevations, must have the depression angle ε subtracted from them. (See Art'y Notes No. 29.)

   This is done once for all for the particular battery and the elevation scales modified accordingly.
2. For atmospheric conditions under which δ_1/δ is not unity, cognizance must be taken of this fact, first by finding the value of δ_1/δ from meteorological conditions observed ; and then by finding the effect produced on the range.
3. For a wind of given velocity and direction, the components must be found in the direction of the range, and across the range ; and the effects on the range must be found and applied.
4. (a) If the muzzle velocity is known to be different from that for which the range-table was computed, and if, moreover, its true value is known, the effect of this change on the range must be found and applied.

   (b) If the muzzle velocity is not known, but if after corrections have been made for all other observed or known abnormalities, it is found that the observed range could have occurred only on the supposition that the velocity was not the same as for the range-table but was a certain other velocity indicated by the range correction board, then the latter is presumed to be the muzzle velocity and allowance made accordingly.

5. Any change in projectile must be taken account of, and its effect on the range applied.
6. The effect of the varying height of the tide must also be considered, as it varies the height of the gun above the water.
7. The travel of the target in the time of flight and also in the prediction interval must be found and allowed for.

So far the corrections indicated are in the direction of the range. Corrections *across* the range are, in our service, made by means of angular deflections.

The following abnormalities will occur:

1. Deflection due to wind, (obtained from anemometer and resolved into components.) The cross-component must be allowed for in the aiming of the gun.
2. Deflection due to travel of target in the time of flight and the prediction interval.
3. Deflection due to drift. This is not strictly an abnormality as it can be computed when the range-table is computed; but it is simpler to consider the normal trajectory as lying altogether in the plane of departure.

These corrections (in range and deflection) are made by the use of suitable devices.*

We will first take up the computation of range-tables and then that of corrections.

As the manner of computing elements has already been outlined in Chapter III, and that of computing C, in Chapter II, it remains only to compute a range-table along these lines.

To illustrate the method we will use the 12″ gun, 2250 f.s.; $w = 1046$; $d = 12$; $c = 1$, and compute the value of the ballistic coefficient for a range 9000 yards and then calculate the remaining elements, including, for the present, only φ, T, ω, $\cot \omega$, y_0, and v_ω.

Designating successive approximations to C by C_1, C_2, etc., and using $C_1 = \frac{w}{d^2}$, we proceed as follows:

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* See Chapter on Gunnery.

$\log w = 3.01953$
a.c. $2 \log d = 7.84164$

$\log C_1 = 0.86117$

$\log X$ 4.43136
a.c. $\log C_1$ 9.13883

$\log Z$ 3.57019
Z 3717

V	A	a_0''	log B′
2200	.0349	1091	.1396
2300	.0318	1090	.1388
2250	.0334	1090	.1392

$\log A$ 8.5237
$\log C_1$ 0.8612

$\log \sin 2\varphi$ 9.3849

2φ 14°02′.4
φ 7°01′.2

$\log a_0''$ 3.0374
$\log C_1$ 0.8612

$\log \tan \varphi$ 9.0904

$\log y_0$ 2.9890
y_0 975
f_a 1.02

$\log f_a$ .00860

$\log B' =$ .1392
$\log \tan \varphi = 9.0904$

$\log \tan \omega$ 9.2296

ω° 9.6
φ° 7.0

$\omega + \varphi$ 16.6

$135 + \omega + \varphi$ 151°.8

$\frac{135 + \varphi + \omega}{135} = 1\frac{1}{8}$ nearly.

log " $= .05115$

$\log \frac{d^3}{2w} = 9.91698$

$\log \gamma = 9.96813$

a.c. $\log \gamma$ .03187
$\log f_a$ .00860
$\log C_1$ .86117

$\log C_2$ .90164

$\log X$ 4.43136
a.c. $\log C_2$ 9.09836

$\log Z$ 3.52972
Z 3386

V	A	a_0''	log B′
2200	.03069	978	.1272
2300	.02798	977	.1263
2250	.02934	978	.1267

$\log A$ 8.46746
$\log C_2$ .90164

$\log \sin 2\varphi$ 9.36910

2φ 13°31′.8
φ 6°45′.9
φ° 6°.77

$135 + \omega + \varphi$ 150.80

log " 2.17840
a.c. log 135 7.86967

$\log \frac{d^3}{2w}$ 9.91698

$\log \gamma$ 9.96505

$\log a_0''$ 2.99034
$\log C_2$ .90164

$\log \tan \varphi$ 9.07418

$\log y_0$ 2.96616
y_0 926
f_a 1.0185

$\log B'$ 9.0742
$\log \tan \varphi$ .1267

$\log \tan \omega$ 9.2009

ω° 9°.03
$\omega^\circ + \varphi^\circ = 15.80$
a.c. $\log \gamma$ .03495

$\log f_a$ .00796
$\log C_1$ .86117

$\log C_3$ .90408

Comparing these values of log C, viz.:

log C_1	.86117
log C_2	.90164
log C_3	.90408

we see that C is rapidly steadying, and should correct once more to see if there is any material change.

log X	4.43136
a.c. log C_3	9.09592
log Z	3.52728
Z	3367

V	A	a_0''	log B′
2200	.03046	971	.1265
2300	.02776	971	.1255
2250	.02911	971	.1260

log A	8.46404	log a_0''	2.98722	log B′	.1260
log C_3	.90408	log C_3	.90408	log tan φ	9.0732
		log tan φ	9.07320		
log sin 2φ	9.36812			log tan ω	9.1992
2φ	13°29′.9	log y_0	2.96450	ω	8°59′.4
φ	6°45′.0	y_0	921.6	ω°	8°.99
φ°	6°.75	f_a	1.0184	φ°	6.75
$135 + \omega + \varphi$	150.74	log f_a	.00792	$\varphi^\circ + \omega^\circ$	15.74
log "	2.17823	a.c. log γ	.03512		
a.c. log 135	7.86967	log C_1	.86117		
log $\frac{d^3}{2w}$	9.91698	log C_4	.90421		
log γ	9.96488				

No further correction of C is now necessary, and we may compute the values of φ, T, ω, cot ω, v_ω and y_0 for 9000 yards range, as follows:

X = 27000 feet log C = .90421

log X	4.43136
a.c. log C	9.09579
log Z	3.52715
Z	3366

V	A	log B′	u	T′	a_0''
2200	.03046	.1265	1420	1.915	973
2300	.02776	.1255	1490	1.827	970
2250	.02911	.1260	1455	1.871	972

log A	8.46404	log B′	.1260	log a_0''	2.98767
log C	.90421	log tan φ	9.0732	log C	.90421
log sin 2φ	9.36825	log tan ω	= 9.1992	log tan φ	9.07320
2φ	13°30′.1	ω	= 8°59′.4	log y_0	2.96508
φ	6°45.0	log cot ω	= .8008	y_0	923
		cot ω	= 6.32		

$$\begin{aligned} \log u &= 3.16286 \\ \log \cos \varphi &= 9.99698 \\ \log \sec \omega &= .00538 \\ \hline \log v_\omega &= 3.16522 \\ v_\omega &= 1463 \end{aligned}$$

$$\begin{aligned} &\log C && .90421 \\ &\log T' && .27207 \\ &\log \sec \varphi && .00302 \\ \hline &\log T && 1.17930 \\ &T && 15.11 \end{aligned}$$

Ex. 1.—Calculate log C for this same gun, velocity and projectile for 3000, 6000 and 12000 yards, and find φ, T, ω, y_0 v_ω in each case. For results (except y_0) see data of Ex. 1, page 30.

Ex. 2.—Using these and the data for 9000 yards find by interpolation* values of φ, T, ω, y_0 and v_ω for each thousand yards from 0 to 12000.

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* See Appendix III.

CHAPTER V.

RANGE CORRECTIONS.

These will be considered in the following order:

1. Those due to changes in atmospheric conditions, and those due to changes in muzzle velocity for any cause except a change in weight of projectile.
2. Those due to a change in weight of projectile.
3. Those due to wind.
4. Those due to tide and travel of target.

Changes due to V *and* C.

As the correction is to be applied on a range-correction chart it must be found from the range-finder range. This range is acquired by the shot under the actual (abnormal) conditions. We must plot any corrections opposite the corrected range, but we must use the actual range as a basis. If, then, with assumed abnormal conditions we compute the angle of departure necessary to reach this range, we will, if we find what range this angle of departure would have given us under normal or range-table conditions, have found that range at which to set the elevation range scale *which is in yards and represents normal conditions*. Too strong emphasis cannot be laid on the fact that the ranges on the elevation range-scales are but the representatives of angles of departure.

It is evident that, if a correction be applied to the actual range-finder range, so that the range setting on the elevation scale will give the correct angle of departure, we have solved the problem.

This correction having been found it is applied through the medium of the range-correction board, and the plotting board.

Changes in the Ballistic Coefficient.

We habitually calculate the changes in range due to a change at intervals of 10% in the ballistic coefficient as follows:

$$-20\%, \quad -10\%, \quad +10\%, \quad +20\%.$$

Using for illustration the values already computed for C and the muzzle velocity and range used, viz.: $V = 2250$; Range 9000 yards; $\log C = 0.90421$ we proceed to find these corrections as follows:

We first find $\frac{X}{C} = Z$ (normal conditions).

Then $\frac{X}{C + \triangle C} = Z_1$ (actual conditions).

with Z_1 take out A_1 for $V =$ muzzle velocity of range table. The φ needed to produce this range under actual conditions *could* be calculated and would be such that

$$\sin 2\varphi_1 = A_1 (C + \triangle C).$$

But we wish to find the range which this φ would produce under range table conditions. In that case A (which call A_2) would have to be such that

$$\sin 2\varphi_2 = A_2 C = A_1 (C + \triangle C) \therefore A_2 = A_1 (1 + \triangle C/C)$$

With A_2 take out from Table II, the corresponding value of Z (call it Z_2). This is $\frac{X_2}{C}$; whence X_2.

$$\text{Then} \quad X - X_2 = \triangle X \text{ feet}$$

$$\text{and} \quad \frac{X - X_2}{3} = \triangle R \text{ yards.}$$

To illustrate with the data quoted above; viz.:

$$V = 2250;\ X = 27000 \text{ feet};\ \log C = 0.90421.$$

Plan, as above, for -20%, say,:

$$\frac{X}{C} = 3366 = Z;\quad \frac{Z}{.80} = 4207.5 = Z_1;$$

$$Z_1 \text{ gives } A_1 = .04149 \quad \text{for } V = 2200.$$

Hence $A_2 = .04149 \times .80 = .03319$ and the corresponding $Z_2 = 3587.3$.

Similarly, for $V = 2300$

$$A_1 = .03780$$
$$A_2 = .03024$$
$$Z_2 = 3586$$

Hence Z_2, for $V = 2250$, is $3587-$.

Now since Z and Z_2 both have the normal C in the denominator we have

$$\triangle R = \frac{\triangle X}{3} = \frac{Z - Z_2}{Z} \times \frac{X}{3} = -591 \text{ yards.}$$

The work is best arranged as follows:

$$\begin{aligned} \log X &= 4.43136 \\ \text{a.c.} \log C &= 9.09579 \\ \hline \log Z &= 3.52715 \\ Z &= 3366 \end{aligned}$$

$V = 2200$

	-20%	-10%	$+10\%$	$+20\%$
Z_1	4207.5	3740	3060	2805
A_1	.04149	.03515	.02685	.02401
A_2	.03139	.03164	.02954	.02881
Z_2	3587	3463	3291	3229

$V = 2300$

	-20%	-10%	$+10\%$	$+20\%$
Z_1	4207.5	3740	3060	2805
A_1	.03780	.03203	.02448	.02189
A_2	.03024	.02883	.02693	.02627
Z_2	3586	3463	3290	3229

$V = 2250$

	-20%	-10%	$+10\%$	$+20\%$
Z_2	3587	3463	3291	3229
$Z - Z_2$	-221	-97	$+75$	$+137$
$\log Z - Z_2$	2.34439_n	1.98677_n	1.87506	2.13672
$\log \frac{X}{3Z}$	.42713	.42713	.42713	.42813
$\log \triangle R$	2.77152_n	2.41390_n	2.30219	2.56385
$\triangle R$	-591	-259	$+200$	$+366$
Corrected R	9591	9259	8800	8634

(b) *Changes in the Muzzle Velocity.*

Taking the neighboring tabular muzzle velocities we use $\triangle V = -150, -50, +50, +150$. We find the abnormal V's to be 2100, 2200, 2300, 2400 f.s.

Proceed as follows :

$Z = 3366$

Abnormal V	=	2100	2200	2300	2400
$A_2 = A_1$	=	.03354	.03045	.02776	.02539
$(V = 2200)$; Z_2	=	3615	3366	3139	2930
$(V = 2300)$; Z_2	=	3867	3604	3365	3146
$(V = 2250)$; Z_2	=	3741	3485	3252	3038
$\therefore\ Z - Z_2$	=	-375	-110	114	328
$\log (Z - Z_2)$	=	2.57403_n	2.07554_n	2.05690	2.51587
$\log \frac{X}{3R}$	=	.42713	.42713	.42713	.42713
$\log \triangle R$	=	3.00116_n	2.50267_n	2.48403	2.94300
$\triangle R$	=	-1002	-318	$+305$	$+877$
Corrected R		10002	9318	8695	8123

The process employed is briefly explained as follows:

Take the actual Z (C not changing), and find for the abnormal V the proper A for this Z. This A or A_1 is the same as A_2, since C does not change; this A_2 will give Z_2 for $V = 2250$ giving $X_2 = CZ_2$ which is the range due under range-table conditions to an angle of departure φ_1 such that $\sin 2\varphi_1 = A_1C = A_2C$. Hence, neither C nor φ changing, we find

$$\frac{X - X_2}{3} = \Delta R.$$

The process is analagous to that employed under (a).

2. *Change in Weight of Projectile.*

(a) The supposition is made that the projectile is changed in weight no alteration being made in the charge.

We then have a change in the muzzle velocity and a change in the ballistic coefficient.

Taking the following formulas from Interior Ballistics, viz:

$$\frac{\Delta V}{V} = -\tfrac{7}{16}\frac{\Delta w}{w}$$

also

$$\frac{\Delta C}{C} = \frac{\Delta w}{w}$$

we see that for small changes in w (as is generally the case for slight change in weight of projectile) we have

$$\frac{\Delta R}{R} = \frac{\Delta w}{w}\left\{ a - \tfrac{7}{16} b \right\}.$$

in which a and b are the values of the ratio of $\frac{\Delta R}{R}$ to $\frac{\Delta C}{C}$ and to $\frac{\Delta V}{V}$ respectively.

They are found thus: (taking for illustration the 12-inch gun at 9000 yards).

For a change of 50 f. s. in V we find an average range change $\frac{305 + 318}{2} = 312$ yards.

Hence $$\frac{312}{9000} = b \times \frac{50}{2250}$$

or $$b = \frac{312}{200} = 1.56$$

For a change of 10 per cent in C we have an average range change of

$$\frac{259 + 200}{2} = 230$$

Hence
$$\frac{9000}{230} = \frac{a}{10}$$
$$\text{or } a = .256$$

It is also true that $a = 1 - \dfrac{\tan \varphi}{\tan \omega} = 1 - \dfrac{1}{B'}$. In this case $\log B' = .1260$ and $\log \dfrac{1}{B'} = 9.8740$.

Hence $$a = 1 - .748 = .252$$

The slight difference is due to calculation with the ballistic tables by interpolation.

The latter value is the better, though the difference is unimportant.

a and b should be calculated for each range at intervals of 1000 yards.

(b) If the charge is changed to give the range-table V with the new projectile, we have

$$\frac{\Delta V}{V} = 0, \text{ and the formula becomes}$$

$$\frac{\Delta R}{R} = a\,\frac{\Delta C}{C} = a\,.\,\frac{\Delta w}{w}.$$

(c) In order to change a muzzle velocity by ΔV f.s. (small changes) we use the formula for nitrocellulose powder

$$\frac{\Delta V}{V} = \tfrac{6}{5}\,\frac{\Delta \tilde{\omega}}{\tilde{\omega}} \quad \therefore \quad \frac{\Delta R}{R} = \tfrac{6}{5}\,b\,\frac{\Delta \tilde{\omega}}{\tilde{\omega}}$$

For nitroglycerin powder we have

$$\frac{\Delta R}{R} = \tfrac{4}{5}\,b\,\frac{\Delta \tilde{\omega}}{\tilde{\omega}}$$

3. *Range Corrections for Wind.*

The formula for wind conditions may be found as follows: We have for a W_x-mile wind component,

$$\frac{\Delta C}{C} = f_w - 1 = \pm\,\frac{2\,W_x T^{\frac{5}{4}}}{X} = \pm\,\tfrac{2}{3}\,.\,\frac{WT^{\frac{5}{4}}}{R}$$

To illustrate

$$T = 15^s.11\,; \qquad R = 9000$$

Hence

$$\frac{\Delta C}{C} = \pm\,\tfrac{2}{3}\,.\,\frac{(15.11)^{\frac{5}{4}}}{9000} = \pm\,\frac{60\,W_x}{27000} = \pm\,\frac{W_x}{450}$$

4. *Corrections due to Tide and Travel of Target.*

(a) Tide is taken account of by the slope of fall. The correction is gotten by taking the change in height of tide and multiplying it by the cotangent of the angle of fall. This gives the correction in feet and ⅓ of this gives it in yards.

Thus, h being the height of tide above mean low water we have

$$\Delta R = \frac{h}{3} \cot \omega$$

Thus for a tide 10 feet above mean low water, we have for 9000 yds. range for 12″ gun; V = 2250 f. s.

$$\Delta R = \frac{10}{3} \times 6.32 = \frac{h}{3} \cot \omega = 21 \text{ yds.}$$

(b) Travel of target.

We are here concerned with the travel of the target in the prediction interval and the time of flight based on its observed travel during the prediction interval.

The following is self-explanatory:

Range 9000 yds; V = 2250 f. s.; T = 15.11 sec.

$$\text{Travel during time of flight} = T \times \frac{\text{obs. travel}}{\text{obs. interval}}$$

Observed travel	Observing interval (seconds)		
yds.	15	20	30
100	101 yds.	76 yds.	50 yds.
200	201 "	151 "	101 "
300	302 "	227 "	151 "

Note.—The corrections obtained by the methods given above are applied through a graphic chart, and the form of this chart is affected by the travel corrections.

It is clear that, in order to use the principle of similar triangles we must make the time of flight scale equicrescent, and this will make the range scale non-equicrescent.

The scale on which the string is set (corresponding to a certain prediction interval) must be graduated so that when set at a certain travel the string will cover the same travel opposite the time of flight which is equal to the prediction interval. We frequently find two or more such scales on the chart for different prediction intervals.

Interpolation

The values for the changes in C and V supply a means of finding by interpolation the range corrections for each 2%

change in C and for each 10 f.s. change in V; and thence the corrected range R′. The method of interpolation given in Appendix III is applied. (See pp. 92-105)

To illustrate with the variations in C, V, and W_x for 9000 yards range with the 12-inch gun V = 2250, w = 1046 pounds, we have

1. Changes in C.

$\frac{\Delta C}{C}$ %	ΔR yds.	δ_1	δ_2
−20	−591	+332	−73
−10	−259	+259	−59
0	0	+200	−33
+10	+200	+166	
+20	+366		

From −20% to zero and from zero to +20% we have, since m = 5, interpolating to 2% intervals,

	−20% to zero	zero to +20%
A	66.40	40.00
2B	− 2.92	− 1.36
A−4B	+72.24	42.72

The successive differences are

	+72.24	+42.72	
	69.32	41.36	
	66.40	40.00	
	63.48	38.64	
	60.56	37.28	
	57.64	35.92	
	54.72	34.56	
	51.80	33.20	
	48.88	31.84	
	45.96	30.48	
Sum	591.00	366.00	check.

Applying these differences we find range corrections and corrected ranges as follows:

%	ΔR	R′=R−ΔR	%	ΔR	R′=R−ΔR
−20	−591	9591	+20	+366	8634
−18	−519	9519	18	336	8664
−16	−449	9449	16	304	8696
−14	−383	9383	14	270	8730
−12	−320	9320	12	236	8764
−10	−259	9259	10	200	8800
−08	−201	9201	08	163	8837
−06	−147	9147	06	124	8876
−04	− 95	9095	04	84	8916
−02	− 46	9046	02	43	8957

2. Changes in V.

ΔV f.s.	ΔR yds	δ_1	δ_2	δ_3
—150	—1002	+684	—61	+10
— 50	— 318	+623	—51	
+ 50	+ 305	+572	(—41)	
+150	+ 877			

(The second difference —41 is gotten from the assumption that δ_3 is constant. Its use will be seen).

To interpolate to 10 f.s., it is best to use three sets; viz:

	—150 to —50	—50 to +50	+50 to +150
A	+68.4	+62.3	+57.2
B	— 0.305	— 0.255	— 0.205
2B	— 0.610	— 0.510	— 0.410
A—9B	+71.145	+64.595	+59.045
Differences in succession	70.535	64.085	58.635
	69.925	63.575	58.225
	69.315	63.065	57.815
	68.705	62.555	57.405
	68.095	62.045	56.995
	67.485	61.535	56.585
	66.875	61.025	56.175
	66.265	60.515	55.765
	65.655	60.005	55.355
Total	684.000 = δ_1	623.000	572.000 check

Applying these differences in succession, we find,

ΔV	ΔR	R′	ΔV	ΔR	R′
—150	—1002	10002	+150	+877	8123
—140	— 932	9932	140	822	8178
—130	— 861	9861	130	766	8234
—120	— 791	9791	120	710	8290
—110	— 722	9722	110	653	8347
—100	— 653	9653	100	596	8404
— 90	— 585	9585	90	539	8461
— 80	— 518	9518	80	481	8519
— 70	— 451	9451	70	423	8577
— 60	— 384	9384	60	364	8636
— 50	— 318	9318	50	305	8695
— 40	— 253	9253	40	245	8755
— 30	— 189	9189	30	184	8816
— 20	— 126	9126	20	123	8877
— 10	— 63	9063	10	62	8938

3. Wind effect.

First find the equivalent percentage change in C and then find by interpolation the change in range incident thereto, using the table of values of ΔR corresponding to each 2% of $\frac{\Delta C}{C}$.

To illustrate with the same range and gun as before, we have

$$\frac{\Delta C}{C} = f_w - 1 = \frac{2W_x T^{\frac{5}{4}}}{X} = \frac{W_x}{450} = 2\tfrac{2}{9}\% \text{ per 10 miles.}$$

For this, we tabulate as follows:

W_x	$\frac{\Delta C}{C}$	ΔR	R'
—50 (head)	—.1111	—294	9294
—40	—.0889	—227	9227
—30	—.0667	—165	9165
—20	—.0444	—105	9105
—10	—.0222	— 51	9051
0	±.0000	0	9000
+10	+.0222	+ 47	8953
+20	+.0444	+ 93	8907
+30	+.0666	+137	8863
+40	+.0889	+180	8820
+50 (rear)	+.1111	+220	8780

The corrected ranges R′ are given in each case, as it is desired to plot each correction opposite the corrected range since in the present system the corrected range is necessarily the argument of the range correction chart, the plotting board giving corrected ranges, while the correction itself must be based on the true range.

Ex. 1. Given the following sets of values:

V	R (yds.)	φ	T	ω	v_ω
2250	3000	1°47′.6	$4^s.27$	1°58′	1973 f.s.
	6000	3°58′.6	$9^s.27$	4°47′	1708
	9000	6°45′.0	$15^s.11$	9°00′	1463
	12000	10°26′.9	$22^s.35$	15°24′	1261

(a) Find A in each case for range-table conditions.

(b) Determine a and b by the use of +10% and —10% change in C and +50 f.s. and —50 f.s. change in V.

(c) Find range corrections for $\frac{\Delta C}{C}$ and ΔV for use in construction of a range-correction chart, for every 10% from 80% to 120%, and for every 100 f.s. from 2100 to 2400 f.s., and find corrected range.

(d) Find $\frac{\triangle C}{C}$ for +50 and − 50 miles per hour.

~~miles per hour.~~

(e) Find corrections for height of tide, −10, −5; +5 and +10 feet.

(f) Find corrections for travel ±100, ±200, ±300 yards in each of the intervals 15, 20 and 30 seconds.

Ex. 2.

(a) Find $\triangle R$ for corrected ranges 3000, 6000, 9000, 12000 yards using data found trom (c) and (d) of Ex. 1, and the method of Appendix III. Application II.

(b) Interpolate to 2% of C, 10 f.s. of V and 10 mi. hr. of W_x.

(c) Be prepared to make a range correction chart for this gun (12-inch $V = 2250$ f. s.)

CHAPTER VI.

DEFLECTION CORRECTIONS—PERFORATIONS

I. Deflection Corrections

1. Wind.

The formula used for this purpose is deduced as follows:

A projectile on emerging from a piece with a horizontal velocity $V \cos \varphi$ enters an atmosphere moving across the plane of fire at the rate of W_z miles per hour or $\frac{44}{30} W_z$ feet per second.

Place $\frac{44}{30} W_z = W$ so as to represent the wind velocity in feet.

We have, then, an angle between the projectile's velocity relatively to the wind and the projectile's horizontal velocity such that its tangent is $W / V \cos \varphi$. The projectile is travelling initially at this angle relatively to the wind. If this angle were maintained it would have travelled contrary to the wind in a range X feet, a distance $X \times \dfrac{W}{V \cos \varphi}$. But since it travels laterally against the wind it suffers retardation due to the resistance it encounters. The effect of this retardation may be thus found:

The effect of the retardation on the range is, in feet,

$$V\,T \cos \varphi - X$$

and the ratio of the range retardation to the lateral retardation is the ratio of the two velocities, since the time of flight is the same. Representing this ratio by tan α, we find

$$(V\,T \cos \varphi - X) \tan \alpha = W\,T - \frac{W\,X}{V \cos \varphi}$$

$$\text{or } \tan \alpha = \frac{W}{V \cos \varphi} \left\{ \frac{V\,T \cos \varphi - X}{V\,T \cos \varphi - X} \right\} = \frac{W}{V \cos \varphi}$$

Hence $\tan \alpha = \dfrac{W}{V \cos \varphi}$ or the initial angle is maintained.

This renders the solution of the problem simple.

The wind would carry the projectile if the latter travelled with it (and at the same speed), $W\,T$ feet for a range X; or its deflection would be, in radian measure

$$\frac{W\,T}{X}$$

The radian measure of α is, clearly,

$$\frac{W}{V \cos \varphi}$$

The difference is the wind deflection in radians, or

$$\text{Deflection (radians)} = \frac{\mathrm{W\,T}}{\mathrm{X}} - \frac{\mathrm{W}}{\mathrm{V}\cos\varphi}$$

$$= \frac{\mathrm{W}}{\mathrm{V}\cos\varphi}\left\{\frac{\mathrm{V\,T}\cos\varphi}{\mathrm{X}} - 1\right\}$$

Replacing W by $\frac{44}{30}\mathrm{W}_y$ remembering that a radian is $\frac{180}{\pi}$ degrees, and taking $\pi = \frac{22}{7}$ we have

$$\text{Deflection (degrees)*} = \tfrac{44}{30} \times 180 \times \tfrac{7}{22} \times \frac{\mathrm{W}_y}{\mathrm{V}\cos\varphi}\left\{\frac{\mathrm{V\,T}\cos\varphi}{\mathrm{X}} - 1\right\}$$

$$= \frac{84\,\mathrm{W}_y}{\mathrm{V}\cos\varphi}\left\{\frac{\mathrm{V\,T}\cos\varphi}{\mathrm{X}} - 1\right\}.$$

Ex. Find deflection for 10 mile wind for 3000, 6000, 9000 and 12,000 yards from values of V, φ, T and R already given for 12″ gun, V = 2250 f.s.

2. Drift.

The deflection due to drift is given by the formula

$$\text{Drift (degrees)} = (1 - \mathrm{K})\frac{\mathrm{d}^3}{\mathrm{wn}}(\varphi^\circ + \omega^\circ)\sec\varphi.$$

Take K for shot or steel shell, as 0.75 and for C. I. shell as 0.80.

(See Journal U. S. Artillery, July–August, 1907, for derivation of this formula).

Ex. Find the drift for 3000, 6000, 9000, 12,000 yards for the 12″ gun, V = 2250 f.s., for shot and A. P. shell.†

PERFORATION

Formulas for perforation of average K. C. armor, by capped projectiles:—

For v = 1800 f.s. and under

$$\frac{\mathrm{t}}{\mathrm{d}} = 0.3677 \times \frac{\mathrm{v}}{1000} \times \frac{\mathrm{v} + 400}{1000} \times \sqrt{\frac{\mathrm{w}}{\mathrm{d}^3}}$$

For v = 1800 f.s. and over

$$\frac{\mathrm{t}}{\mathrm{d}} = \frac{9}{7}\sqrt{\frac{\mathrm{w}}{\mathrm{d}^3}} \times \left\{\frac{\mathrm{v}}{1000} - \frac{2}{3}\right\}.$$

These formulas agree for v = 1800 f.s.

For oblique impact, α being the angle of incidence (measured from the normal to the plate) the following table of percentages to be *subtracted* from perforations at normal impact for each value of a, is given

---

* See Appendix IV.

† See Appendix IX.

α °	%	α °	%
0	0	25	6
5	0	30	8
10	1	35	11
15	2	40	15
20	4	45	19

These formulas give results sensibly the same as those obtained from the curves of Fig. 2, opposite p. 144, Journal U. S. Artillery, March-April, 1907, from which the perforations of the 8″, 10″ and 12″ range tables were derived.

In using these formulas for a particular range table place them in the form

$$\frac{t}{d} = P_v \cdot \sqrt{\frac{w}{d^3}}$$

in which P_v is

below 1800 f.s. $\quad 0.3677 \cdot \frac{v}{1000} \cdot \frac{v + 400}{1000}$.

and above 1800 f.s. $\quad \frac{9}{7}\left\{\frac{v-667}{1000}\right\}$.

We may thus tabulate P_v with v as argument; then

$$\frac{t}{d} = P_v \cdot \sqrt{\frac{w}{d^3}}.$$

TABLE OF VALUES OF P_v.

v	P_v	$\triangle$	v	P_v	$\triangle$
100	.018	26	1600	1.177	136
200	.044	33	1700	1.313	144
300	.077	41	1800	1.457	128
400	.118	48	1900	1.585	129
500	.166	55	2000	1.714	128
600	.221	62	2100	1.842	129
700	.283	70	2200	1 971	129
800	.353	77	2300	2.100	128
900	.430	85	2400	2.228	129
1000	.515	92	2500	2.357	128
1100	.607	99	2600	2.485	129
1200	.706	107	2700	2.614	128
1300	.813	114	2800	2.742	129
1400	.927	121	2900	2.871	129
1500	1.048	129	3000	3.000	

Ex. To find perforation of 12″ gun at 9000 yards, we have

$$\sqrt{\frac{w}{d^3}} = \frac{7}{9}\ ;\ v_\omega = 1463$$

and hence $P_v = .927 + 0.63\left\{.121\right\} = 1.003.$

Hence $$\frac{t}{12} = \frac{7}{9} \times 1.003 = .78$$

and $t = 9.36$ in.

NOTE.—It is sufficient in dealing with the 6-in. 106-lb. projectile to take $\sqrt{\frac{w}{d^3}}$ as 7/10. It is in fact .7005. In dealing with the 8-in., 10-in., 12-in. and 14-in. guns, its value is with satisfactory exactness 7/9, though the numbers are respectlvely .7856; .7772; 7797; 7777.

Using 7/10 we have for the 6-in. gun

$$t = \frac{7d}{10} \cdot P_v$$

and using 7/9 for the others

$$t = \frac{7d}{9} \cdot P_v$$

Ex. Find the perforations in K. C. armor at normal impact and at 30° with the normal for the 12-in. gun, $V = 2250$, $w = 1046$ lbs. at ranges, 0, 3000, 6000, 9000 and 12000 yards from data given in Ex. 1, p. 30.

CHAPTER VII.

HIGH ANGLE FIRE.

Mortar range-tables in our service are prepared from actual firings. These firings are made under carefully recorded conditions of weather, etc., and the muzzle velocities used in making the firings should extend from below the lowest to above the highest muzzle-velocity to be used in service with the particular projectile.

Among the data recorded are the following :

1. Projectile, kind and weight.
2. Charge, kind, lot and weight.
3. Quadrant elevation.
4. Muzzle velocity and breech-block pressure.
5. Range (observed by several observers).
6. Observed deflection in degrees (checked by several observations).
7. Time of flight (several observers).
8. Wind, direction and velocity.
9. Barometer, thermometer and humidity.

In order to obtain the range-velocity-elevation relations necessary as a basis of a range-table, the zones of which will rarely be the same as those used in the firings, corrections must be applied to the ranges observed under firing conditions in order to find out what the true ranges would be under range-table conditions.

The formulas for correction of observed ranges are

1. For change in δ_1/δ or in C,

$$\frac{\Delta X}{X} = \frac{\Delta C}{C}\left\{1 - \frac{g X}{V^2 \sin 2\varphi}\right\} = \frac{\Delta R}{R}$$

2. For a wind-component, W_x miles per hour.

$$\frac{\Delta C}{C} = \pm \frac{2 W_x T^{\frac{5}{4}}}{X} = \pm \frac{2}{3} \frac{W_x T^{\frac{5}{4}}}{R}$$

which, in combination with the formula of 1, gives

$$\Delta X = \pm 2 W_x T^{\frac{5}{4}} \left\{1 - \frac{g X}{V^2 \sin 2\varphi}\right\} \text{ feet.}$$

or

$$\Delta R = \pm \tfrac{2}{3} W_x T^{\frac{5}{4}} \left\{1 - \frac{g X}{V^2 \sin 2\varphi}\right\} \text{ yards.}$$

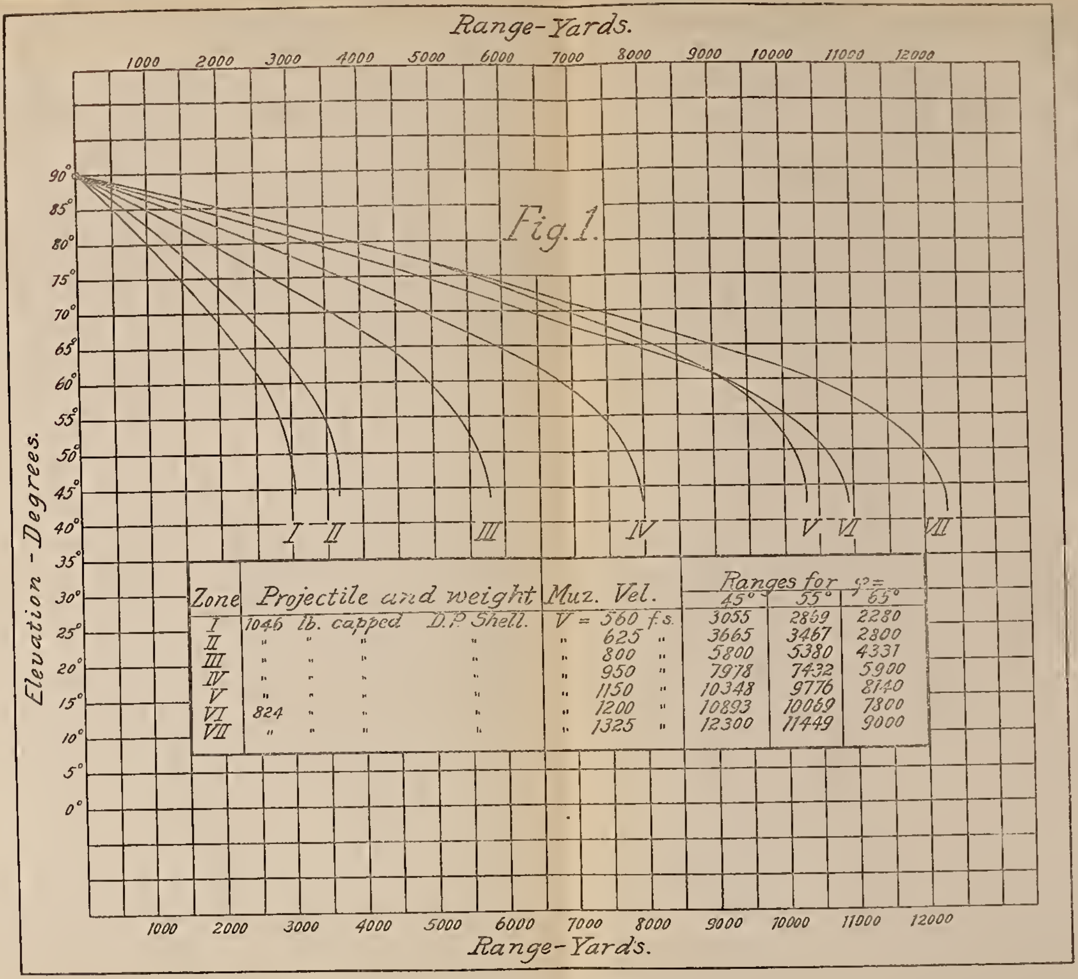
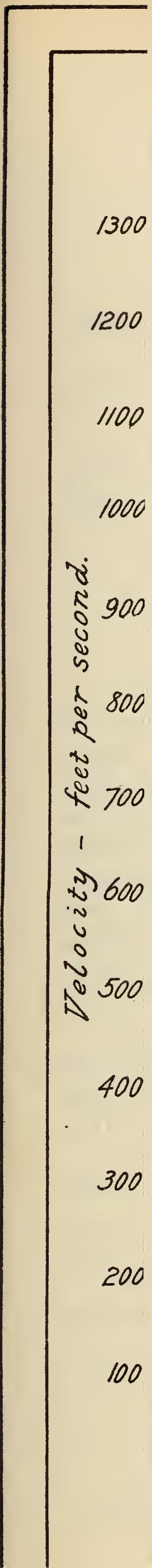


Fig. 1.

Zone	Projectile and weight	Muz. Vel.	Ranges for φ = 45°	55°	65°
I	1046 lb. capped D.P. Shell.	V = 560 f.s.	3055	2869	2280
II	" " " "	" 625 "	3665	3467	2800
III	" " " "	" 800 "	5800	5380	4331
IV	" " " "	" 950 "	7978	7432	5900
V	" " " "	" 1150 "	10348	9776	8140
VI	824 " " "	" 1200 "	10893	10069	7800
VII	" " " "	" 1325 "	12300	11449	9000

Firing Zones
Range-Elevation Curves.

W.C.S.


Velocity - feet per second.
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100

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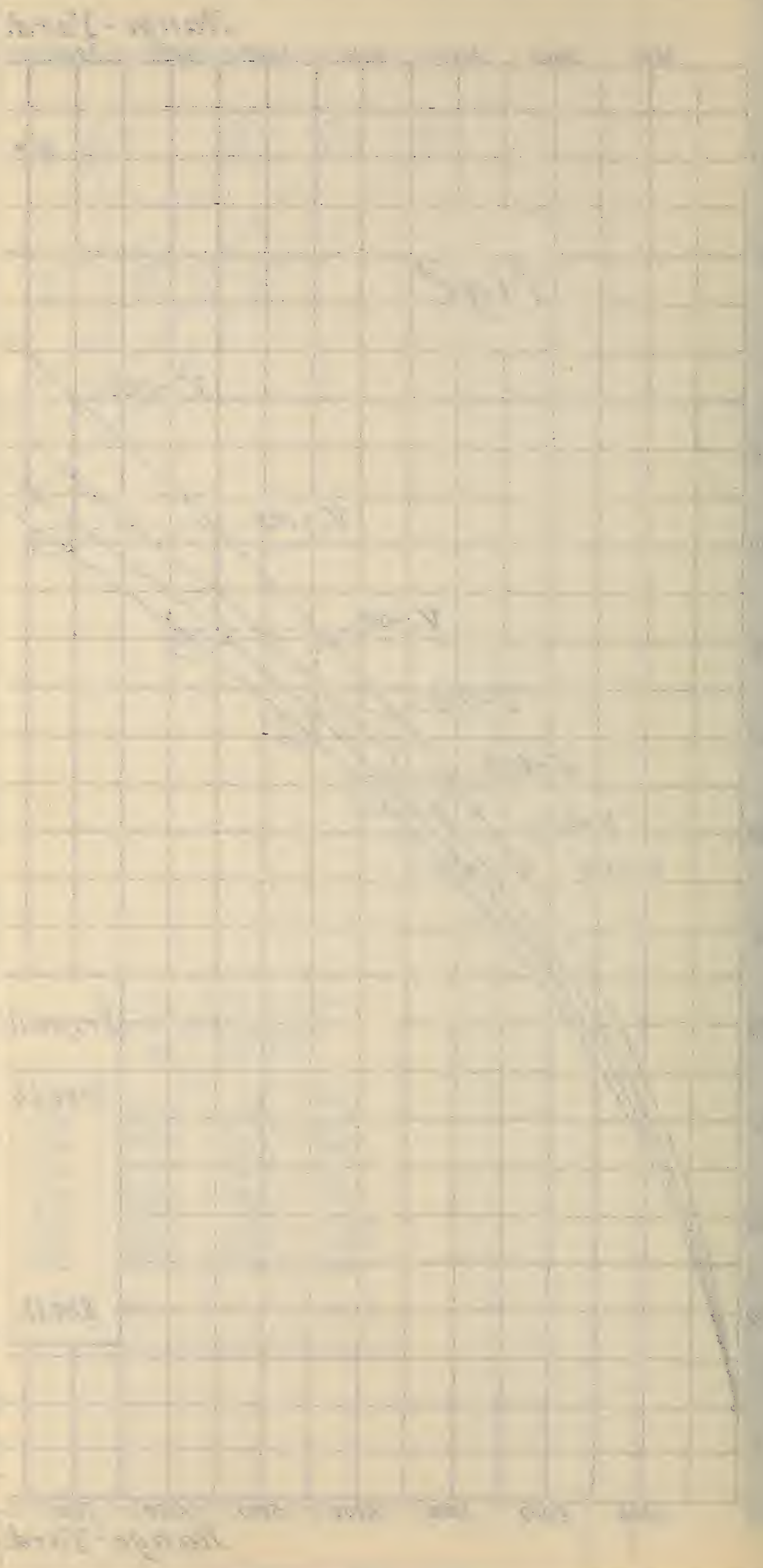

3. For a change in V.

$$\frac{\Delta X}{X} = \frac{2\,g\,X}{V^2 \sin 2\varphi} \cdot \frac{\Delta V}{V} = \frac{\Delta R}{R}$$

In these formulas, X is the range in feet, W_x the wind component in miles per hour and T the time of flight in seconds.*

The ranges given in the firing data in yards will be represented by R, so that

$$X = 3\,R.$$

Exercise: Using the formulas of 1 and 2, find the range correction to be applied to the observed range in the following data

$V = 800$; $W_x = +\,8$ mi. hr.; $\varphi = 45°$; Range 5832 yds.; $T = 34$ sec.; $\delta_{\prime\prime}/\delta = 0.964$.

Results: Wind effect = 55 yds.; atmosphere effect — 27 yds. Total effect + 28 yds.; correction — 28 yds. Corrected range 5804 yds.

After the changes in range are made the ranges are plotted on cross section paper to correspond to the values of φ with which they were obtained, one curve being thus drawn for each firing zone. Great care should be taken in drawing the curves, and much study should be devoted to the conditions under which each shot was fired. The curve should be smooth and of the general shape of those shown in Figure 1. After these curves have been drawn, the values of the range for each 5° of elevation are taken from the curves of Figure 1, and plotted as in Figure 2, which gives a velocity-range curve for each value of φ.

In the two figures (which are very much smaller than would have been the case in practice) only the curves for 45°, 55°, 60° and 65° are given. It is best to put them in for every 5 degrees in the actual drawing.

The double check afforded by the two sets of curves, enables one to place great reliance on the results; and, from them to find the ranges for each elevation for any zone-velocities that may be chosen, using Figure 2.

The zone-velocities selected for the range table will have to fulfil as far as may be the following conditions:

1. Steadiness of flight must be preserved and hence V cannot be taken below a certain value.
2. Consistently with 1, the inferior limit of range should be not greater than a certain range, to avoid an interior dead space.

---

* See Appendix VIII.

3. The overlaps of the zones should not fall below certain widths.
4. The field of fire must be covered effectively to the extreme range desired.
5. The breech-block pressure should not exceed the safe limit.

With the 12-inch steel mortar we have the following conditions: * Lower limit as near 2000 as possible. Lowest velocity for steadiness about 550 f.s. Overlaps about 400 yards and where projectile is changed, an additional hundred or two hundred yards. Outer range about 12,000 yards. The pressure should not exceed about 32,000 lbs. per sq. in. Figure 2 gives the zones determined as best fulfilling these conditions for this mortar.

As an illustration of the use of Figure 2, with a curve for every 5° of φ from 45° to 65°, the following ranges, were taken from the complete set of curves (after interpolating by differences) for 810 f.s., 12-inch mortar, 1046 lb. projectile,

φ	Range (yds.)	△	φ	Range (yds.)	△
45°	5940	— 15	56°	5439	— 84
46	5925	— 21	57	5355	— 90
47	5904	— 27	58	5265	— 97
48	5877	— 33	59	5168	—104
49	5844	- 39	60	5064	—111
50	5805	— 45	61	4953	—119
51	5760	— 51	62	4834	—127
52	5709	— 58	63	4707	—135
53	5651	— 64	64	4572	—143
54	5587	— 71	65	4429	
55	5516	— 77			

The △ column gives the change in range for a change of 1° of φ; it would indicate a maximum range for an elevation of 42½°; as the first difference vanishes for that value of φ.

Exercise:

Given the following corrected data for the 75-mm. sub-caliber gun, 15-lb. projectile, construct the range-elevation curves, and the range-velocity curves and from them find the muzzle velocities for three zones with an inner range of 2200 yards and a uniform overlap of 200 yards.

Data:

---

* See Appendix V.

	V		
φ	500 f. s.	600 f. s.	700 f. s.
45°	2200 yds.	3070 yds.	4010 yds.
50	2160	3010	3925
55	2050	2860	3730
60	1870	2620	3425
65	1620	2290	3010

Scale: 1 in. = 500 yds. = 5° = 100 f.s.

APPLICATIONS TO 12″ MORTAR (1046 lb. PROJECTILE).

The correction formulas already quoted for High Angle Fire for changes in V, C, and for wind, sometimes assume simple mean forms, without sacrifice of practical accuracy, for particular mortars and projectiles.

By examining Figure 2, ranges may be tabulated for every 10° of φ from 45° to 65°, for each of the velocities 500, 600, etc., to 1000 f.s. By subtraction we may find the range-differences for 100 f.s. change in V and tabulate these for the same φ's as before and for V's the means of those the change from one to the other of which produces the change in range; that is for 550, 650, 750, etc.

Dividing these range changes by the means of the ranges of which the changes are the differences we find values of $\frac{\Delta R}{R}$ for 100 f.s. change in V as follows:

Values of $\frac{\Delta R}{R}$ for $\Delta V = 100$ f.s.

φ	550	650	750	850	950
45°	.31	.28	.25	.22	.18
55	.31	.27	.24	.23	.18
65	.32	.28	.24	.21	.18
Mean	.313	.273	.243	.220	.180

A change in V of 100 f.s. thus produces a percentage change in the range which is practically independent of the elevation, but dependent on the velocity alone.

To find the relation of $\frac{\Delta R}{R}$ to $\frac{\Delta V}{V}$, we have $\frac{\Delta V}{V} = \frac{100}{V}$ in the above table and $\frac{\Delta V}{V}$ has for values, respectively, $\frac{2}{11}, \frac{2}{13}, \frac{2}{15}, \frac{2}{17}, \frac{2}{19}$.

Hence placing $\frac{\Delta R}{R} = K\,\frac{\Delta V}{V}$ we find

$$\Sigma \frac{\Delta R}{R} = 1.23\,; \qquad \Sigma \frac{\Delta V}{V} = .692$$

$$\text{Hence } K = \frac{1.23}{.692} = \frac{16}{9}$$

Hence for this mortar and projectile

$$\frac{\Delta R}{R} = \frac{16}{9} \cdot \frac{\Delta V}{V}$$

But the general formula gives, for a change in V,

$$\frac{\Delta R}{R} = \frac{2gX}{V^2 \sin 2\varphi} \cdot \frac{\Delta V}{V}$$

Hence we may replace $\frac{2gX}{V^2 \sin 2\varphi}$ by $\frac{16}{9}$.

This, of course, is only an average value of $\frac{2gX}{V^2 \sin 2\varphi}$ but serves to show the general coincidence between the mean values thus derived and the mean of the values obtained from a strict calculation of $\frac{V^2 \sin 2\varphi}{gX}$ for each V, as will now be shown.

We find from the data that

$$\frac{V^2 \sin 2\varphi}{gX} = 1 + 0.235\left(\frac{V}{1000}\right)^2$$

represents with practical exactness the value of $\frac{V^2 \sin 2\varphi}{gX}$ *for use with the correction formulas.* It gives values as follows:—

V	$1 - \frac{gX}{V^2 \sin 2\phi}$	$\frac{2gX}{V^2 \sin 2\phi}$
500	.055	1.89
600	.080	1.84
700	.105	1.79
800	.130	1.74
900	.160	1.68
1000	.190	1.62

The average value 1.76 of $\frac{2gX}{V^2 \sin 2\varphi}$ between the values for V = 500 and V = 1000 is thus a check on the average value just found. The values given in this table for each velocity should be used except for quick and simple calculations regarding *changes in range for a changed V,* in which latter case the mean value $\frac{16}{9}$ may be used. For ΔC or wind use values of first column.

An example of the use to which the approximate ratio $\frac{gX}{V^2 \sin 2\varphi} = 8/9$ may be put is as follows:—

The 12-inch steel mortar range table gives the values of the range for 45° and 65° for each zone. These are tabulated below with the corresponding muzzle velocity; and also the mean of the ranges at 45° and 65°, with the elevation to the nearest degree, which, according to the range table, gives the latter range; thus:

Zone.	V	Ranges at 45°	Ranges at 65°	Mean range.	Corresponding elevation.
					°
I.	550	2970	2210	2590	59
II.	600	3431	2600	3015	59
III.	660	4030	3070	3550	59
IV.	725	4800	3631	4216	59
V.	810	5940	4429	5185	59
VI.	915	7476	5520	6498	59
VII.	1050	9250	7027	8139	59

It thus appears that the mean range of each zone occurs at about 59°. This is a still further indication that the ratio $\frac{V^2 \sin 2\varphi}{gX}$ is practically independent of the elevation with this heavy mortar, since, V being constant, X, under the above supposition, would be proportional to sin 2φ and hence the average of the ranges at 45° and 65° would occur at an elevation such that

$$\sin 2\varphi = \frac{\sin 2 \times 65° + \sin 2 \times 45°}{2}$$

$$= \tfrac{1}{2} \left\{ 1.000 + 0.766 \right\} = .883 = \sin 62°$$

From which

$$2\varphi = 180° - 62° = 118°, \text{ or } \varphi = 59°.$$

If, then, $\frac{\Delta R}{R} = \frac{16}{9} \frac{\Delta V}{V}$, we have, on the supposition that the diminution in velocity due to a change in the ballistic qualities of the powder, is proportional to the velocity, the following guide in correcting ranges in zones from trial shots in a single zone:

The trial shot should be fired preferably at 59°. The *percentage* change in the range found necessary to allow for the change in velocity should be applied in all the zones.

Thus, if shots were fired at 59° in Zone V. and the mean point of impact were 519 yards short, the percentage correction is + 10% or + 519 yards for that zone and is easily found for them all, in order, to be

I. + 259; II. + 302; III. + 355; IV. + 422; V. + 519; VI. + 650; VII. +814 yards.

These are the mean corrections by sliding the zones in or out bodily, as is customary. It is plain that these corrections are midway between those for 45° and 65° and hence afford better corrections for the zone as a whole, than would the range corrections found by firing at any other elevation than 59°.

CHAPTER VIII.

THE COMPUTATION OF MORTAR RANGE TABLES.

After the establishment of zone-velocities and ranges at 5° intervals, it remains merely to calculate the remaining elements of the range-table, viz.: T, v_ω, drift (in degrees), angle of fall and perforation in deck-armor; and to interpolate results to single degrees.

These are found through the medium of the table of High Angle Fire Ratios,* in the following manner. With the assumed V, φ, and X from the curves of Fig. 2 (for example $V = 810$ f.s.; $\varphi = 45°$; Range $= 5940$ yds.) take out from the proper φ-page (in this case 45°) the values of $\frac{10\,T}{V}$, ω, $\frac{v_\omega}{V}$, using $\frac{10X}{V^2} = \frac{30R}{V^2}$ as argument; ω is thus determined outright and T and v_ω may be readily computed by multiplying the ratios by $\frac{V}{10}$ and V respectively. Thus:

$$\frac{10X}{V^2} = \frac{30 \times 5940}{(810)^2} = .272 -$$

Corresponding to $\frac{10X}{V^2} = .272$, we find $\frac{10T}{V} = .424$; $\omega = 48°\ 26'$; $\frac{v_\omega}{V} = .881$; also $\frac{y_0}{X}$; $\frac{V}{\sqrt{C}}$ and $\log \frac{h}{X}$ if the latter three are desired.

From the first three we have $T = 81 \times .424 = 34.34$ sec. $\omega = 48\frac{1}{2}°$; $v_\omega = 810 \times .881 = 714$ f.s.

If desired, $y_0 = .266 \times 3 \times 5940$ ft. $= 4740$ ft.; $\log \frac{h}{X} = 9.224$ whence $\frac{h}{X} = .1675$ and $h = .1675 \times 3 \times 5940 = 2985$ ft. This is the mean height of the trajectory.*

PERFORATION.

Perforations are found by the use of the formula for deck-steel:

$$t^2 = [0.835 - 10] \frac{w\,v_\omega{}^3}{d} \times F$$

in which $F = \sec^3 \frac{a}{2} \cos^2 a$ (a being $90° - \omega$).

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* See Appendix VIII.

A table of values of log F, F, $\sqrt{F}$ and $1-\sqrt{F}$ is given below:

ω °	log F -10	$\triangle$	F	$\triangle$	$\sqrt{F}$	$1-\sqrt{F}$
45	9.802	.048	.63	.08	.80	.20
50	9.850	.038	.71	.06	.84	.16
55	9.888	.032	.77	.06	.88	.12
60	9.920	.026	.83	.05	.91	.09
65	9.946	.020	.88	.05	.94	.06
70	9.966	.015	.93	.03	.96	.04
75	9.981	.011	.96	02	.98	.02
80	9.992	.006	.98	.02	.99	.00
85	9.998	.002	1.00	.00	1.00	.00
90	0.000		1.00		1.00	.00

The column $\sqrt{F}$ is the ratio of the oblique thickness to the normal thickness, and taking the mean terminal velocity for the zone and calculating the perforation for normal impact this factor will give the perforation for any angle of fall.

The column $1-\sqrt{F}$ gives the percentage loss of thickness on account of the oblique impact.

Hence the perforation for normal impact may be computed from the mean v_ω for the zone (v_ω is sensibly constant in a zone) and the percentages indicated in the column $1-\sqrt{F}$ subtracted for the proper ω.

Ex. Find t where v_ω, ω, w and d are given as follows:

$$w = 1046\,;\quad v_\omega = 714 \text{ f.s.};\quad \omega = 48\tfrac{1}{2}^\circ.$$

Here $\log F = 9.802 + \frac{3.5}{5} \times .048 = 9.836-10$

Hence

$$\begin{aligned}
\text{Const. log} &= 0.836-10 \\
\log W &= 3.020 \\
3 \log v &= 8.561 \\
\text{a.c. } \log d &= 8.921 \\
\log F &= 9.836-10 \\
\hline
2 \log t &= 1.174 \\
\log t &= .587 \\
t &= 3.9 \text{ in.}
\end{aligned}$$

DRIFT.

The drift is computed from the formula

$$\text{Drift (degrees)} = \frac{d^3}{wn}(1-K)\, 2\varphi \sec\varphi$$

For C.I. shell $K = \frac{4}{5}$; for D.P. shell $K = \frac{3}{4}$

Hence for C.I. shell

$$\text{Drift}^\circ = \frac{2}{5}\frac{d^3}{wn}\ \varphi \sec \varphi$$

For D.P. shell

$$\text{Drift}^\circ = \frac{1}{2} \cdot \frac{d^3}{wn} \cdot \varphi \sec \varphi.$$

This formula for drift as well as that for direct fire assumes the ogive struck with a radius of 2 calibers and projectiles of the usual form and length. For any other, the coefficient $(1 - K)$ must be found by actual firings.

The values of the coefficient of $\varphi \sec \varphi$ can be computed from the data for the mortar and projectile. Thus, for 1046 lb. projectile, $\frac{d^3}{w} = 1.652$ and for the 824 lb., $\frac{d^3}{w} = 2.097$. These may be taken as $\frac{5}{3}$ and $\frac{21}{10}$ without material error.

The twist (1 turn in n calibers) is represented in the formula by $1/n$; n is 20 for the steel mortar and 25 for the C.I. steel hooped.

Hence we have the following values for the coefficient

$$2\frac{d^3}{wn}(1-K).$$

Mortar.	n	Projectile, weight and kind.			
		1046		824	
		D.P.	C.I.	D.P.	C.I.
C. I.	25	.0330	.0264	.0419	.0336
Steel	20	.0413	.0330	.0524	.0419

Hence, practically, the coefficient of $\varphi \sec \varphi$ is as shown for the following combinations:

I	1046 D.P. and steel mortar 824 C.I. and steel mortar 824 D.P. and C.I. mortar	$\frac{1}{24} = .04167$
II	1046 D.P. and C.I. mortar 1046 C.I. and steel mortar 824 C.I. and C.I. mortar	$\frac{1}{30} = .0333$
III	1046 lb. C.I. and C.I. mortar	$\frac{2}{75} = .0266$

It will be noted that these coefficients are in the ratio $\frac{1}{24} : \frac{1}{30} : \frac{2}{75}$ or 25:20:16.

Hence using as a basis $\frac{1}{30}$ (Class II) we may take $\frac{\varphi \sec \varphi}{30}$, and then $\frac{5}{4}$ and $\frac{4}{5}$ of this respectively for classes I and III.

The values of φ sec φ and of $\frac{1}{30}$ φ sec φ are given in the following table:

Values of φ sec φ and of $\frac{\varphi \sec \varphi}{30}$

φ	φ sec φ	$\frac{1}{30}$ (φ sec φ)	φ	φ sec φ	$\frac{1}{30}$ φ sec φ
45	63°.64	2°.121	55	95°.89	3°.196
46	66 .22	2 .207	56	100 .15	3 .338
47	68 .92	2 .297	57	104 .66	3 .489
48	71 .74	2 .391	58	109 .45	3 .648
49	74 .69	2 .490	59	114 .55	3 .818
50	77 .79	2 .593	60	120 .00	4 .000
51	81 .04	2 .701	61	125 .82	4 .194
52	84 .46	2 .815	62	132 .06	4 .402
53	88 .07	2 .936	63	138 .77	4 .626
54	91 .87	3 .062	64	146 .00	4 .867
55	95 .89	3 .196	65	153 .80	5 .127

For the sub-caliber gun C.I. shell, $n = 29.83$; $K = .814$; $d = 2.95275 = 75^{\text{mm}}$; $w = 18$; and we have a coefficient $\frac{1}{56}$ instead of $\frac{1}{30}$; for the 15 lb. projectile of this same gun we have a coefficient $\frac{18}{15}$ ($= 1.2$) times the above or $\frac{3}{140}$.

(See Article on Drift, Journal U. S. Artillery, July-Aug., 1907, or Appendix IX.)

WIND DEFLECTIONS

Wind deflections are calculated by the use of the formula, already deduced, viz.:

$$\text{Deflection (degrees)} = \frac{84\ W_y}{V \cos\varphi} \left\{ \frac{VT \cos\varphi}{X} - 1 \right\}$$

This takes a more convenient form for High Angle Fire as follows:

$$\text{Deflection (degrees)} = \frac{84\ W_y}{V} \left\{ \frac{V\ T}{X} - \sec\varphi \right\}.$$

$$= \frac{84\ W_y}{V} \left\{ \frac{V\ T}{3\ R} - \sec\varphi \right\}.$$

Ex. 1. Calculate the drift in degrees and hundredths for each combination mentioned above for φ at 5° intervals from 45° to 65°.

2. (a) Calculate perforation in inches of deck armor for the following:

$d = 12''$; $w = 1046$ lbs.

φ	ω						
45°	46°	47°	48°	48°	48°	49°	50°
55	56	57	57	58	58	58	60
65	66	67	67	67	68	68	69
v_ω	523	560	602	652	720	807	884
M. V.	550	600	660	735	810	915	1050

(b) From the results plot on cross-section paper separately the perforations for the 1046 lb. D.P. projectile as a function of the muzzle velocity for elevations 45°, 55°, 65°, (three curves), axis of v horizontal. Scales: vertical, 1 in. = 1 in.; horizontal 1 in. = 100 f.s. Begin with $V = 550$ f.s. and $t = 2.4$ in.

3. Find the deflections due to a 10-mile wind for each of the following, and compare results.

75^{mm} (a) $V = 700$; $\varphi = 65°$; Range 2968 yds.; $T = 37.1$ sec.
$12''$ (b) $V = 600$; $\varphi = 65°$; Range 3410 yds.; $T = 33.8$ sec.

CHAPTER IX.

GUNNERY.

Gunnery in its widest sense embraces a knowledge of Exterior and Interior Ballistics and the use of *Artillery Material*, the object of the art and science involved being the destruction of or maximum damage to the object at which artillery fire is directed.

This chapter will, however, be restricted to the application of ballistic data to practical use at the battery, and to a description of the general character of the auxiliary instruments by which the range of the target is found and the gun directed so as to hit the target.

1. For every type of gun, carriage, and projectile, there should be certain firings, called *range-firings*, conducted with a view to determining the peculiarities of gun, mount, and projectile. This having been done, a range-table may be computed, and range- and deflection-corrections found.

2. The elevations given in the range-table referred to must be corrected by subtracting the absolute value of the position angle, ϵ, due to height of trunnions above mean low water and to curvature, before they are applied to the range-scale of any gun. This applies only to guns using direct fire. The range-table elevations are not altered for mortars.

3. The ranges and the corresponding elevations having been tabulated for each gun (see Artillery Notes, No. 29) may be applied to the range-scale of the gun by the use of a clinometer, the gun being given (by means of the clinometer) the elevations pertaining to ranges at fixed intervals (say 200 yards) and the *range* is marked on the range-scale in place of the elevation. *The ranges of the range-scale thus represent elevations under range-table conditions.* (See p. 22; Part I).

Ordinarily these graduations are applied under the supervision of the Ordnance Department to the metal of the range-scale. If, however, conditions should arise under which it has to be done by the personnel of the battery the graduations should be made on a scale made of durable paper applied to the surface of the range-scale by an adhesive which does not greatly affect the dimensions of the paper, and, *after it has set* and the paper is firmly in place the scale should be graduated by use of the clinometer as already described. The scale should then be varnished with shellac to prevent injury or distortion by moisture. The same precaution against moisture should be observed in the use of paper scales in any portion of the equipment of a battery.

The adjustment of the gun for elevation is now complete until calibration firings show further adjustment in elevation to be necessary, due to peculiarities of the individual gun and mount.

The adjustment of the sight for deflections, is readily made by causing the vertical hair of the sight to bisect the same object as that bisected by bore sights, the object being stationary and at a range not less than half the extreme range-table range of the gun, the sight being set for "no deflection".

4. The gun and carriage having been thus adjusted to range-table conditions, it is necessary to have some means of finding the range of the target, its rate of travel, and the corrections in elevation and deflection necessary in order to allow for the motion of the target and for variations from range-table conditions.

The means of adapting range-table data as applied to a battery to conditions actually obtaining at the time of firing constitute collectively

THE POSITION FINDING SERVICE.

This consists of

1. *The Position Finder.*
2. *The Plotting Board.*
3. *The Range Board.*
4. *The Deflection Board.*
5. *The Atmosphere Chart.*
6. *The Wind Component Indicator.*
7. *The Powder Chart.*

These will be briefly described in the order given.

1. Position Finders.

Position finders, properly so called, are used for finding, at any instant, the *position* of a target ; that is, its *range and azimuth.* They are thus distinguished from range-finders which merely give the range. Range-finders may be used with the secondary armament but a position-finder is needed with a heavy battery for reasons which will appear.

Position-finders are of two principal kinds : Horizontal and vertical (or depression) position-finders.

a. A horizontal position-finder consists essentially of two azimuth measuring instruments, one at each end of a base-line, and having a telephone connection and a time interval bell to synchronize the observations. The position of the target at fixed intervals is thus determined by the two azimuths. A map may thus be made on the plotting board of the path of the target showing its position at uniformly spaced times.

b. A depression position-finder consists of only one instrument and is thus self-contained or under the complete control of a single individual. By it are found the range and azimuth of the target at fixed intervals of time.

The vertical position-finder mechanically solves a triangle in a vertical plane, after correcting for the conditions of tide curvature and atmospheric refraction that actually obtain.

It may be said that the vertical position-finder, by means of a triangle within itself and similar to the right triangle formed by the height of the instrument above the water (corrected for refraction and curvature), the true horizontal range, and the straight line from the instrument to the water-line of the target, gives at any time, from a scale, the range in yards; and the azimuth is given by the azimuth measuring feature of the instrument.

The correction for height of tide above mean low water is accomplished in the instrument by an adjustment independent of the range, since the tide is assumed to be the same over the field of fire.

Curvature of the earth varies with the range, and may be readily computed as in Appendix VII. Atmospheric refraction is the least stable and least uniformly acting abnormality with which the D.P.F. has to contend. *Normal* refraction is assumed quite generally to counteract $\frac{1}{7}$ of curvature; that is to say, by assuming the earth to have only $\frac{6}{7}$ of its actual curvature we may, once for all, allow for *normal* refraction.

But if the refraction be *abnormal* (or different from its assumed value), which is usually the case, this arrangement fails as an exact solution, since the correction for curvature and refraction depends on the range.

The general solution of the problem would appear to be the combination of refraction and curvature into one effect represented in the instrument by a similar curve, the curve to be able to adapt itself to actual conditions, as determined by making the instrument read true on all the important datum points in the field of fire. Of course conditions of refraction are frequently temporary and readjustment from time to time will be necessary.

In some forms of instrument the correction is made once for all in the instrument for *curvature and normal refraction*, and any other state as to refraction is allowed for by adjusting the tide-scale, thereby making the instrument read approximately true on two datum points.

The ranges and directions of the target at fixed time-intervals are thus obtained from the position-finder (of whatever kind) and sent to the plotting room where the position of the target is continually plotted on a plotting board.

2. *The Plotting Board.*

This device consists essentially of a drawing board provided with means of quickly mapping the data sent from the position-finder, finding the rate of travel of the target in range and azimuth for use with the range and deflection correction devices, finding by means of an arm (called the gun arm) the range and azimuth to the target from the directing gun (or point) of the battery, and applying to the true range and azimuth the corrections received from the correction boards.

For direct fire the plotting board takes no cognizance of azimuth corrections which are sent to the gun directly from the deflection correction board, which, however, receives the data as to rate of travel in azimuth from the plotting board.

3. *The Range Board.*

This board applies through mechanical addition and subtraction the information received as to atmospheric conditions, tide, travel of target and variation in muzzle velocity, obtaining a range-correction which when applied through the gun arm of the plotting board gives the artificial or corrected range for use in elevating the gun.

4. *The Deflection Board.*

This combines by mechanical addition and subtraction the deflections due to cross-wind, drift and travel of target, giving a corrected azimuth or deflection.

5. *The Atmosphere Chart.*

This device gives the reference number due to the reading of the barometer and thermometer, for use with the atmosphere correction feature of the range board.

6. *The Wind Component Indicator.*

Information as to the wind velocity and direction having been received from the meteorological station, this device is used to resolve the wind velocity into components across and in the plane of fire, the direction in which the components are to be applied on the range and deflection correction devices being shown by the reference numbers.

7. *The Powder Chart.*

This is a chart showing the muzzle velocity to be expected from a slight change in the weight of the charge and from variations in the temperature of the powder from the standard temperature 70° F., at which the prescribed change should give the muzzle velocity of the range-table.

Reference Numbers.

These are used to avoid the confusion incident to the use of positive and negative numbers.

II. PRACTICE

In our service practice is preceded by trial shots. In the preliminary work of determining conditions the adjustment of all scales should be carefully made and especially the elevation and deflection devices of the individual gun on its mount. All such adjustments having been made and the ammunition being prepared for use in accordance with orders and regulations, the gun should (at the proper time) be loaded, especial pains being taken to properly seat the projectile. This is of great importance, as, with the projectile not properly seated, the powder gas escapes past the projectile and considerable loss of velocity occurs.

As no intelligent correction can be made for such an improper seating it is essential to avoid such a situation by careful drill in loading.

Trial shots are fired giving the gun the proper corrected elevation and deflection for the assumed range which it is expected that the shot will attain. This involves taking into consideration the knowledge, if any, that is on record from prior firings with the same powder as to what muzzle velocity is to be expected from it, as well as the data as to meteorological and other conditions. In making the range-corrections, it is to be remembered that the range used on the range-board as a basis for corrections, is the *corrected range* (see p. 22, Part I). Hence, taking the actual data and the actual supposed range find the range corrections and thence the corrected range ; using this corrected range apply the corrections again until they cease to change. The plotting board now shows the corrected range which is to be sent to the gun. The deflection board gives the proper deflection correction.

The gun may now be fired and the range and deflection of the shot observed. In this manner fire all the trial shots, correcting no further for muzzle velocity, but correcting for other conditions, provided the latter show any change.

Then take the *mean of the observed ranges* as the true range.

To find what the muzzle velocity must have been, take the difference between the expected range and the mean observed range and find from the range-board what diminution or increase must be applied to the velocity as assumed in the trial shots.

In case the first of the trial shots falls very wide of the expected range, so as to interfere with observation of the fall of the shots by parties stationed in the neighborhood of the point of fall, a suitable correction in range should be made and attributed to velocity. Suitable correction having been made in the recorded value of the range of the first trial shot for the difference in velocity between itself and the succeeding ones, it may, *if it was accurately observed*, be incorporated with the succeeding ones in obtaining a mean observed range. *Otherwise not.*

The results of trial shots should be carefully considered and recorded with a description and history of the powder used, for use in estimating muzzle velocity in future trial shots.

The mean of the ranges of the trial shots is taken because as shown by the theory of errors it is safest to do so.

Having found the muzzle velocity, the corrections for the first shot of the practice must be found by first finding the corrected range from the range-finder range as in the case of the first trial shot. For subsequent shots the corrected range of the shot last fired is already known.

III. CALIBRATION.

Calibration, as the term is now used is the process of ascertaining the peculiarities of individual guns of a battery and *correcting the range scale so that each gun ranges correctly* on an average. This enables the battery commander to treat the battery as if it were a single gun when in action or at practice. In calibrating a battery the powder should be such as to give the proper muzzle velocity. In that case the range scales of each gun should be adjusted so as to agree with the range which the calibration test indicates for range table conditions.

For calibration to be of great and certain value, the powder used must be uniform in action, and carefully weighed. It is best when practicable to measure the muzzle velocity for each round, but this is generally impracticable. The elevation should be given by clinometer and verified after loading and before firing. The elevation given should be that due to the range corrected for actual conditions by use of the position finding service. The mean range of each gun should be recorded, also the mean deflection and the mean error of each. The difference from the expected range of the actual mean range for each gun should be properly applied in yards to the range scale of that gun. It is then calibrated, for that range.

For example a gun is laid for an assumed actual range 6000 yards. The tentative corrections based on 6000 yards show a corrected range less than that; using this find new corrections until the corrected range is established before firing. Lay the gun at the corrected range which we will take for illustration to be 5500 yards. If the ranges are observed to be 5870, 5920, 5840, 5850, 5900, the average is 5876. Thus the gun lacks 124 yards of shooting up to the range. Set the range scale now at 6000 yards, and move the scale or pointer until the pointer is opposite 5876 on the scale. The gun is now adjusted in range. If each gun is thus adjusted on its mean range the battery will behave as a unit.

Deflections could be tested in the same way, but they are usually small and unless there is material discrepancy between guns the sights need not be changed, the principal object of calibration being the adjustment of the range scale.

As all our direct fire guns elevate by scale and use the sight only to follow the target in azimuth, the only adjustment of the sight that is necessary is that by which the axis of collimation of the sight is made to cross the axis of the bore at a mean range. This is done by the use of bore sights in the gun and the sight seat is adjusted by the adjusting screws until axis of telescopic sight and line of bore sights bisect the object on which observations are taken. (See also Artillery Notes, No. 22. For mortars see Artillery Notes, Nos. 22 and 11).

IV. ACCURACY.

The accuracy of a battery is determined by the percentage of hits which it may be expected to make on the service target.

It is best determined from the firings used in calibrating. It must be found for each individual gun and then, after the battery is calibrated the errors may be treated individually as errors of the battery. Suppose for instance that of three guns in a battery firing with an assumed actual range 6000 yards, say, the record is as follows :

	Gun No. 1	Gun No. 2	Gun No. 3
	5780	5946	6024
	5820	5910	6085
	5910	5982	5990
	5875	5895	6041
	5850	5947	6000
sum	29235	29680	30140
mean	5847	5936	6028

The pointers should now be set at 6000 on each gun and the *scales or pointers moved* without changing the elevation so that the pointer is opposite the actual mean range reached in each case.

Taking the observed range in each case minus the mean range of the group we have, averaging errors without regard to sign, for guns 1, 2, 3, respectively.

	−67	+10	− 4
	−27	−26	+57
	+63	+46	−38
	+28	−41	+13
	+ 3	+11	−28
sum	188	134	140
mean	±37.6	±26.8	±28.0

These are the mean errors for the individual guns. *After calibration*, their mean is the mean for the battery, thus

$$\begin{array}{r} 37.6 \\ 26.8 \\ 28.0 \\ \hline 3)92.4 \\ \hline 30.8 \end{array}$$

or ±30.8 is the mean error for the battery.

To show how the mean error of a gun or battery may be expected to give a definite idea of the number of hits that may be expected with perfect work on the part of the personnel the following is given :

The mean error is the average of errors without regard to sign. In each case the sum of the negative errors is the same as that of the positive errors, since the mean range is subtracted from the observed range in each case. Positive and negative errors are equally likely to occur. This mean error is the error of average size or the average error having regard to the frequency of occurence of each size of error. Large errors are less frequent than small ones and this plays an important part in the theory of errors. Another error, called the probable error, is such that just half the shots fired will in the long run have an error equal to or less than this probable error. This is gotten from the mean error by multiplying by 0.845. The mean error thus enables us to find the probable error. The probable errors for guns 1, 2, and 3 are, respectively,

31.772; 22.646; and 23.660.

and for the battery when calibrated

26.026

Now since half the shots will in the long run fall over or short of the mean range by amounts not greater than the probable error, and half of them short and half over, it follows that, if we draw two lines perpendicular to the line joining the battery to the mean point of impact of the shots, the two lines being beyond and short of the mean range by a distance equal to the probable error, we will have a zone or belt in which half or 50% of the shots will fall. This is called the 50% zone and the total width of the 50% zone is thus twice the probable error or 1.69 times the mean error.

The width of the 50% zone being taken as the unit of comparison the theory of errors deduces the widths of zones of other percentages in terms of the 50% zone width as a unit.

Thus the factor Z/Z_1 in the table on page 57 shows the width, in terms of that of the 50% zone, of the zone whose percentage is given in the column next on its left.

As an illustration, the 60% zone is 1.25 times as wide as the 50% zone, the 82½% zone is twice as wide. The 99⅓% zone is four times as wide. The zone four times as wide as the 50% zone is called the enveloping zone, since it practically contains all the shots.

To apply this table to the illustration of a battery of three guns which we have used we find the enveloping zones to be for the three guns and the battery, respectively, of widths

254, 181, 189 and 208 yards.

We are mainly concerned however with hitting the standard target of such guns which is 30 feet high and 60 feet wide. We are also concerned chiefly with its height, as errors in deflection are small. The slope of fall at this range we will take to be say 1 on 10. Hence the horizontal equivalent of the vertical target is $10 \times 30 = 300$ feet, and, aim being taken at the water line, this will, if the gun is properly laid (the center of impact on point aimed at) give a space of 300 feet or 100 yards *beyond* the target; allowing half that or 50 yards for ricochet hits by shots falling short, we find that the danger space consists of two *half* zones, one beyond and the other short of the target.

Per cent.	Factor Z/Z_1	Per cent.	Factor Z/Z_1	Per cent.	Factor Z/Z_1	Per cent.	Factor Z/Z_1	Per cent.	Factor Z/Z_1
1	0.02	21	0.40	41	0.80	61	1.27	81	1.94
2	0.04	22	0.41	42	0.82	62	1.30	82	1.98
3	0.06	23	0.43	43	0.84	63	1.33	83	2.03
4	0.07	24	0.45	44	0.86	64	1.36	84	2.08
5	0.09	25	0.47	45	0.89	65	1.39	85	2.13
6	0.11	26	0.49	46	0.91	66	1.42	86	2.18
7	0.13	27	0.51	47	0.93	67	1.45	87	2.24
8	0.15	28	0.53	48	0.95	68	1.48	88	2.30
9	0.17	29	0.55	49	0.98	69	1.51	89	2.37
10	0.18	30	0.57	50	1.00	70	1.54	90	2.44
11	0.20	31	0.59	51	1.02	71	1.57	91	2.52
12	0.22	32	0.61	52	1.04	72	1.60	92	2.60
13	0.24	33	0.63	53	1.07	73	1.64	93	2.69
14	0.26	34	0.65	54	1.09	74	1.67	94	2.78
15	0.28	35	0.67	55	1.12	75	1.71	95	2.91
16	0.30	36	0.70	56	1.14	76	1.74	96	3.04
17	0.32	37	0.72	57	1.17	77	1.78	97	3.22
18	0.34	38	0.74	58	1.19	78	1.82	98	3.45
19	0.36	39	0.76	59	1.22	79	1.86	99	3.82
20	0.38	40	0.78	60	1.25	80	1.90	100	

As our table concerns itself with zones symmetrically disposed with respect to the center of impact (or point aimed at) we must *not* take 100 yards and 50 yards as *zone widths* but as *half zone widths*. Taking, then, 200 and 100 yards as the zone widths we may find the percentages and halve the latter.

Thus for a zone 200 yards wide we find for the battery, the 50% zone width of which is 52 yards

$$\frac{Z}{Z_1} = \frac{200}{52} = 3.84 \quad \therefore \quad \text{per cent} = .99$$

and for the 100 yard zone width

$$\frac{Z}{Z_1} = \frac{100}{52} = 1.92 \quad \therefore \quad \text{per cent} = .80\tfrac{1}{2}$$

Hence, the percentage for the two half zones, taken together, is

$$\frac{.805}{2} + \frac{.990}{2} = \frac{1.795}{2} = 89\tfrac{3}{4}\%$$

or we may expect 89¾% of hits with the calibrated battery; that is 9 hits out of 10 shots. This is the best shooting that can be *required* of the personnel.

It follows that it is important

(a) To adjust all scales and instruments to a condition of minimum error.

(b) To keep a record of all shots fired, with deductions from results, especially from *trial shots*.

(c) To draw correct conclusions from trial shots. It is safest to take the mean range and the mean deflection, unless some definite cause of error existed for a particular shot.

(d) To avoid correction of fire during the practice when the error of a particular shot is within the mean error of the trial shots, or when the difference between two shots is less than the width of the 50% zone.

(e) To exercise the greatest care with the ammunition and verify the weights of projectiles and powder charges and note the condition and temperature of the latter about the time of firing.

(f) To so train the loading detail that the projectile is properly seated, always. This is of great importance, not on account of the density of loading but on account of the escape of gas past the projectile. The bands of the projectiles should be examined beforehand and all burrs and irregularities removed.

Exercises:

1. To fire at a target 6300 yards distant, with the following data, how is the range-scale to be set?

Data:

10-inch rifle, $w = 604$ lbs.; range table V, 2250. Anticipated powder velocity 2250; temperature of powder, 50° Fahr.; temperature of air, 37° Fahr.; barometer, 29.85 inches; wind component (W_x) miles per hour, + 4.0 (rear); powder, International smokeless, lot 6, 1906, 7 days in service magazine.

(a) $\delta_{\prime}/\delta = .974 - .85\,(.974 - .943) = .974 - .85 \times .031 = .948$

Hence

$$\frac{\Delta C}{C} = -\,(1 - 0.948) = -\,.052$$

(See Table A: Values of $\delta_{\prime}/\delta$).

(b) $\triangle V = -33$ f.s. for powder temperature 50° F. (See Table B.)

(c) $W_x = +4$ mi / hr.

I. From these using data for construction of range board (see Table D.) or, preferably, range board itself, based on these data, find the following corrections based on *actual* range 6300 yards,

-5.2% in C,	$+74$ yards
-34 f-s. in V,	$+151$ yards
$+4$, W_x,	-10
Total	$+215$
Corrected range	6515

II. Taking the corrected range as 6500 yards, we find in the same order

$$+78, \ +154, \ -11, = +221 \text{ yards}$$

Corrected range $6300 + 221 = 6521$ yards.

From the range-table for this gun (Table C.) it is seen that this corresponds to an elevation (uncorrected for height of site and curvature) of

$$\begin{aligned} &4^\circ 31'.1 + (4^\circ 42'.3 - 4^\circ 31'.1) \times \tfrac{121}{200} \\ &= 4^\circ 31'.1 + 11'.2 \times .605 \\ &= 4^\circ 37'.9 \end{aligned}$$

The correction for height of site and curvature was that due to a battery 31 feet above mean low water at 6300 yards range or $7'.2$. Hence the clinometer should be set at $4^\circ 37'.9 - 07'.2 = 4^\circ 30'.7$.

Hence set elevation-range-scale, at 6521 yards corresponding to $4^\circ 30'.7$ clinometer elevation.

2. Suppose that as a result of setting and conditions as in 1, the following ranges are recorded, with a particular gun :

Shot No.	Range	Shot No.	Range
1	6190	6	6320
2	6215	7	6235
3	6185	8	6155
4	6290	9	6230
5	6205	10	6230

(a) What is the mean range?

(b) What is the mean error?

(c) What is the probable error?

(d) What is the danger space, $\dfrac{30 \cot(\omega + \varepsilon)}{3} = 10 \cot(\omega + \varepsilon)$

(e) What is the correction to be applied to range-scale?

(f) What is the percentage of hits to be expected on the service target (30 feet high and 60 feet long) including ricochet hits, if the gun is properly laid?

(a) Mean range $= \frac{\Sigma R}{10} = 6225.5 = R_0$

(b) Mean error found as follows

Ranges	Errors $R - R_0$
6190	-35.5
6215	-10.5
6185	-40.5
6290	$+64.5$
6205	-20.5
6320	$+94.5$
6235	$+9.5$
6155	-70.5
6230	$+4.5$
6230	$+4.5$
ΣR 6225.5;	Σe 355.0
R_0 6225.5;	e_0 35.5

That is, mean error is ± 35.5 yards.

(c) Probable error, $r_0 = \pm 35.5 \times 0.845 = \pm 30$ yards.

(d) From range-table $\omega = 5°36'$ and $\omega + \varepsilon = 5°43'$; $\cot(\omega + \varepsilon) = 10.0$ and danger space $= 10 \times 10 = 100$ yds.

(e) Correction to be applied to range scale is

$$6300 - 6225.5 = +74.5 \text{ yards.}$$

(f) The width of the 50% zone is

$$Z_1 = 2 \times r_0 = 2 \times 30 = 60 \text{ yards.}$$

The danger space consists of halves of two zones whose widths are 200 and 100 yards.

Hence

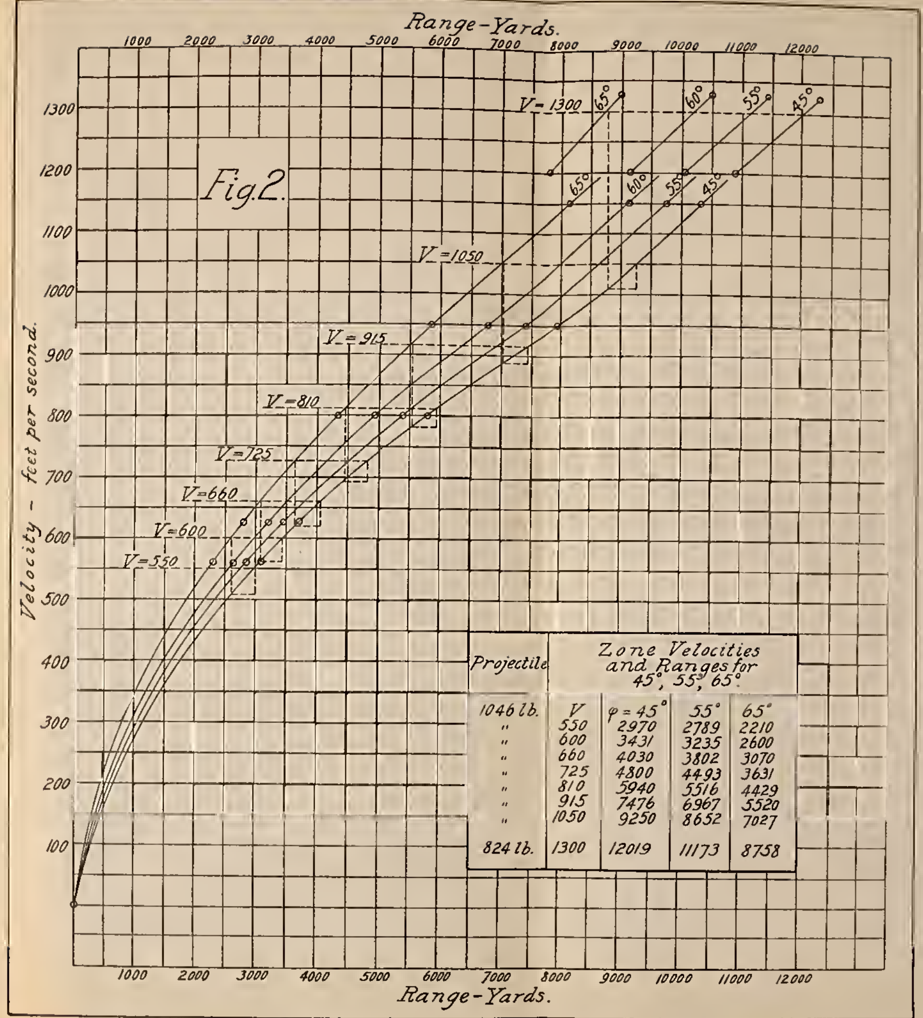
Factors, $\frac{Z}{Z_1}$, in the two cases are $\frac{200}{60}$ and $\frac{100}{60}$ or 3.33 and 1.67.

Hence percentages are .97 and .74 and the mean of these is

$$\frac{1.71}{2} = 85.5\%.$$

Hence this gun if properly laid can be expected to make 17 hits out of 20, in the long run, *at this range.* This is the best that can be *required* of the personnel.

Fig. 2.



Projectile	Zone Velocities and Ranges for 45°, 55°, 65°.			
1046 lb.	V	φ = 45°	55°	65°
″	550	2970	2789	2210
″	600	3431	3235	2600
″	660	4030	3802	3070
″	725	4800	4493	3631
″	810	5940	5516	4429
″	915	7476	6967	5520
″	1050	9250	8652	7027
824 lb.	1300	12019	11173	8758

TABLE B

TABLE OF VALUES OF PERCENTAGE CHANGES IN MUZZLE VELOCITY OF POWDERS TESTED AT 70° F. AND FIRED WITH A MAGAZINE TEMPERATURE t° F.

t	ΔV/V	ΔV for V =		
		2000	2250	2500
0°	—.0323	—65	—73	—81
10°	—.0302	—60	—67	—75
20°	—.0275	—55	—62	—69
30°	—.0241	—48	—54	—60
40°	—.0199	—40	—45	—50
50°	—.0147	—29	—33	—36
60°	—.0082	—16	—18	—20
70°	—.0000	0	0	0
80°	.0101	20	23	25
90°	.0228	46	52	58
100°	.0387	77	87	96

(Computed from the formula

$$\frac{\Delta V}{V} = .00867 \left\{ 2^{.032t} - 2^{.032 \times 70} \right\}$$

$$= .00867 \left\{ 2^{.032t} - 4.73 \right\} .).$$

TABLE A

Values of $\delta_{,}/\delta$ for temperature and pressure of atmosphere 78% saturated with moisture. (From Artillery Note No. 25.)

Ther.	Barometer				Ther.	Barometer.			
F.	28″	29″	30″	31″	F.	28″	29″	30″	31″
—20°	0.890	0.861	0.831	0.806	41°	1.017	0.982	0.951	0.919
—19°	0.892	0.863	0.833	0.808	42°	1.019	0.984	0.953	0.921
—18°	0.894	0.864	0.835	0.809	43°	1.021	0.987	0.955	0.923
—17°	0.896	0.866	0.837	0.811	44°	1.023	0.989	0.957	0.925
—16°	0.898	0.868	0.839	0.813	45°	1.026	0.991	0.959	0.927
—15°	0.901	0.870	0.841	0.815	46°	1.028	0.993	0.961	0.929
—14°	0.903	0.872	0.843	0.816	47°	1.030	0.995	0.963	0.931
—13°	0.905	0.874	0.845	0.818	48°	1.033	0.997	0.964	0.933
—12°	0.907	0.876	0.847	0.820	49°	1.035	0.999	0.966	0.935
—11°	0.910	0.878	0.848	0.822	50°	1.037	1.002	0.968	0.937
—10°	0.912	0.880	0.850	0.824	51°	1.040	1.004	0.970	0.939
— 9°	0.914	0.881	0.852	0.826	52°	1.042	1.006	0.972	0.941
— 8°	0.916	0.883	0.854	0.827	53°	1.044	1.008	0.974	0.943
— 7°	0.918	0.885	0.856	0.829	54°	1.046	1.010	0.976	0.945
— 6°	0.920	0.887	0.858	0.831	55°	1.048	1.012	0.978	0.947
— 5°	0.922	0.889	0.860	0.833	56°	1.050	1.014	0.980	0.949
— 4°	0.924	0.891	0.862	0.835	57°	1.053	1.016	0.982	0.951
— 3°	0.926	0.893	0.864	0.836	58°	1.055	1.018	0.984	0.952
— 2°	0.928	0.895	0.866	0.838	59°	1.057	1.020	0.986	0.954
— 1°	0.930	0.897	0.868	0.840	60°	1.059	1.022	0.988	0.956
0°	0.932	0.899	0.870	0.842	61°	1.062	1.025	0.990	0.958
1°	0.934	0.901	0.971	0.844	62°	1.064	1.027	0.992	0.960
2°	0.936	0.903	0.873	0.845	63°	1.066	1.029	0.994	0.962
3°	0.938	0.905	0.876	0.847	64°	1.068	1.031	0.996	0.964
4°	0.940	0.907	0.878	0.849	65°	1.071	1.033	0.998	0.966
5°	0.942	0.909	0.880	0.851	66°	1.073	1.035	1.001	0.968
6°	0.944	0.911	0.881	0.853	67°	1.075	1.037	1.003	0.970
7°	0.946	0.913	0.883	0.855	68°	1.078	1.040	1.005	0.973
8°	0.948	0.915	0.885	0.856	69°	1.080	1.042	1.007	0.975
9°	0.950	0.917	0.887	0.858	70°	1.082	1.044	1.009	0.977
10°	0.952	0.919	0.889	0.860	71°	1.085	1.046	1.011	0.979
11°	0.954	0.921	0.890	0.862	72°	1.087	1.048	1.013	0.981
12°	0.956	0.923	0.892	0.864	73°	1.089	1.050	1.015	0.983
13°	0.958	0.925	0.894	0.866	74°	1.092	1.053	1.017	0.985
14°	0.960	0.927	0.897	0.867	75°	1.094	1.055	1.019	0.987
15°	0.962	0.929	0.899	0.869	76°	1.096	1.057	1.022	0.989
16°	0.964	0.931	0.901	0.871	77°	1.099	1.059	1.025	0.992
17°	0.966	0.933	0.903	0.873	78°	1.101	1.062	1.027	0.994
18°	0.968	0.935	0.905	0.875	79°	1.104	1.064	1.029	0.996
19°	0.971	0.937	0.907	0.877	80°	1.106	1.066	1.031	0.998
20°	0.973	0.939	0.909	0.879	81°	1.109	1.068	1.033	1.000
21°	0.975	0.941	0.911	0.881	82°	1.111	1.071	1.035	1.002
22°	0.977	0.943	0.912	0.883	83°	1.114	1.074	1.038	1.005
23°	0.979	0.945	0.914	0.885	84°	1.116	1.076	1.041	1.007
24°	0.981	0.947	0.916	0.887	85°	1.119	1.079	1.043	1.009
25°	0.983	0.949	0.918	0.888	86°	1.121	1.081	1.045	1.011
26°	0.985	0.951	0.920	0.890	87°	1.124	1.083	1.047	1.013
27°	0.987	0.953	0.922	0.892	88°	1.126	1.086	1.049	1.016
28°	0.990	0.955	0.924	0.894	89°	1.129	1.089	1.053	1.018
29°	0.992	0.958	0.926	0.896	90°	1.131	1.092	1.055	1.020
30°	0.994	0.960	0.928	0.898	91°	1.134	1.094	1.057	1.022
31°	0.996	0.962	0.930	0.899	92°	1.136	1.096	1.059	1.025
32°	0.998	0.964	0.932	0.902	93°	1.139	1.099	1.062	1.027
33°	1.000	0.966	0.934	0.903	94°	1.142	1.102	1.064	1.029
34°	1.003	0.968	0.936	0.906	95°	1.144	1.105	1.066	1.031
35°	1.005	0.970	0.938	0.907	96°	1.147	1.107	1.068	1.033
36°	1.007	0.972	0.940	0.909	97°	1.149	1.110	1.071	1.035
37°	1.009	0.974	0.943	0.911	98°	1.152	1.112	1.074	1.037
38°	1.011	0.976	0.945	0.913	99°	1.155	1.115	1.076	1.040
39°	1.013	0.978	0.947	0.915	100°	1.157	1.117	1.079	1.042
40°	1.015	0.980	0.949	0.917					

TABLE C

RANGE TABLE FOR 10-INCH RIFLE

Muzzle velocity:—2250 f. s. *Projectile (capped):—604 lbs.*

Range.	Angle of departure.	Change in elevation for 10 yards in range.	Time of flight.	Angle of fall.	Slope of fall.	Maximum ordinate.	Striking velocity.	Perforation of Krupp armor, capped projectile,		Deflection for—		Ballistic coefficient.
								Impact.				
								Normal.	30° with norm'l	Drift.	Wind component of 10 miles per hour.	
Yards.	° ′	′	*Seconds.*	° ′	*1 on—*	*Feet.*	*f. s.*	*Inches.*	*Inches.*	*Degrees.*	*Degrees.*	*Log. C.*
1000	0 34.1	0.4	1.37	0 35	97.3	8	2136	14.5	13.3	0.019	0.010	0.85922
1200	0 41.3	0.4	1.65	0 43	80.5	12	2114			.023	.012	
1400	0 48.5	0.4	1.94	0 51	67.9	16	2092			.027	.014	
1600	0 55.8	0.4	2.23	0 59	58.4	21	2070			.031	.016	
1800	1 03.2	0.4	2.52	1 07	51.1	26	2048			.035	.018	
2000	1 10.7	0.4	2.81	1 15	45.4	32	2026	13.4	12.2	.040	.021	0.85574
2200	1 18.3	0.4	3.11	1 24	40.8	39	2005			.044	.023	
2400	1 26.0	0.4	3.41	1 33	37.0	47	1983			.049	.025	
2600	1 33.9	0.4	3.72	1 42	33.8	56	1961			.054	.027	
2800	1 41.9	0.4	4.03	1 52	31.0	66	1940			.059	.030	
3000	1 50.0	0.4	4.34	2 02	28.5	76	1918	12.3	11.3	.064	.032	0.85177
3200	1 58.2	0.4	4.66	2 12	26.3	88	1897			.069	.034	
3400	2 06.6	0.4	4.98	2 22	24.3	100	1875			.074	.037	
3600	2 15.1	0.4	5.30	2 33	22.5	114	1854			.079	.039	
3800	2 23.7	0.4	5.63	2 44	20.9	128	1833			.085	.041	
4000	2 32.5	0.5	5.96	2 56	19.5	143	1812	11.2	10.2	.091	.044	0.84733
4200	2 41.5	0.5	6.30	3 08	18.3	159	1791			.097	.046	
4400	2 50.6	0.5	6.64	3 20	17.2	177	1770			.103	.049	
4600	2 59.8	0.5	6.99	3 33	16.2	196	1749			.109	.051	
4800	3 09.2	0.5	7.34	3 46	15.3	216	1728			.115	.054	
5000	3 18.8	0.5	7.69	3 59	14.4	238	1708	10.1	9.2	.121	.056	0.84247
5200	3 28.6	0.5	8.05	4 13	13.6	261	1688			.127	.059	
5400	3 38.6	0.5	8.41	4 27	12.9	285	1668			.134	.061	
5600	3 48.7	0.5	8.78	4 42	12.2	311	1648			.141	.064	
5800	3 59.0	6.5	9.15	4 57	11.6	338	1628			.148	.067	
6000	4 09.5	0.5	9.53	5 12	11.0	366	1608	9.1	8.4	.155	.070	0.83724
6200	4 20.2	0.5	9.91	5 28	10.4	396	1588			.162	.073	
6400	4 31.1	0.5	10.30	5 45	9.9	428	1569			.170	.076	
6600	4 42.3	0.6	10.69	6 02	9.4	462	1550			.178	.079	
6800	4 53.7	0.6	11.09	6 20	9.0	497	1531			.186	.082	
7000	5 05.4	0.6	11.50	6 38	8.6	534	1512	8.2	7.5	.194	.085	0.83167
7200	5 17.8	0.6	11.91	6 57	8.2	573	1493			.203	.088	
7400	5 29.4	0.6	12.33	7 16	7.8	614	1475			.212	.091	
7600	5 41.8	0.6	12.76	7 36	7.5	657	1457			.221	.094	
7800	5 54.5	0.7	13.19	7 57	7.2	703	1439			.230	.098	
8000	6 07.5	0.7	13.63	8 19	6.9	751	1421	7.4	6.8	.240	.101	0.82572
8200	6 20.8	0.7	14.08	8 41	6.6	801	1403			.250	.104	
8400	6 34.4	0.7	14.53	9 04	6.3	854	1386			.260	.108	
8600	6 48.3	0.7	14.99	9 28	6.0	910	1369			.271	.111	
8800	7 02.5	0.7	15.46	9 53	5.7	968	1352			.282	.115	
9000	7 17.0	0.7	15.93	10 19	5.5	1029	1335	6.6	6.1	.294	.118	0.81922
9200	7 31.9	0.8	16.41	10 46	5.2	1093	1319			.306	.122	
9400	7 47.1	0.8	16.90	11 14	5.0	1160	1303			.318	.125	
9600	8 02.7	0.8	17.40	11 43	4.8	1231	1288			.331	.129	
9800	8 18.7	0.8	17.91	12 12	4.6	1306	1273			.344	.133	
10000	8 35.2	0.8	18.43	12 42	4.4	1384	1258	5.9	5.5	.357	.137	0.81190
10200	8 52.1	0.9	18.96	13 13	4.2	1466	1244			.371	.141	
10400	9 09.4	0.9	19.50	13 45	4.0	1552	1230			.385	.145	
10600	9 27.2	0.9	20.05	14 18	3.9	1642	1217			.400	.149	
10800	9 45.5	0.9	20.61	14 52	3.7	1737	1204			.415	.153	
11000	10 04.4	1.0	21.18	15 27	3.6	1836	1192	5.4	5.0	.430	.157	0.80345
11200	10 24.0	1.0	21.76	16 03	3.4	1940	1180			.446	.161	
11400	10 44.3	1.0	22.36	16 40	3.3	2049	1169			.463	.166	
11600	11 05.3	1.1	22.97	17 19	3.2	2164	1159			.480	.170	
11800	11 27.0	1.1	23.59	17 59	3.1	2285	1149			.498	.174	
12000	11 49.4	1.1	24.22	18 40	3.0	2411	1140	5.0	4.6	.516	.179	0.79351

3. If the other gun of the battery when laid at the same elevation gave a mean range of 6086 yards and a mean error ± 101 yards,

(a) What correction should be made in the range elevation-scale?
(b) What is the probable error of this gun?
(c) What is the probable error of the calibrated battery?
(d) What percentage of hits may be expected with this battery, as calibrated?

(a) 6300—6086 = + 214 yards.
(b) ± 101 × .845 = ± 85.3 yards.
(c) $\frac{85.3 + 30}{2} = \pm 58$ yards.
(d) Factors Z / Z_1 for battery as calibrated are $\frac{200}{116}$ and $\frac{100}{116}$

Hence percentage is $\frac{75.3 + 44}{2} = 60\%$ or 6 hits out of 10.

That is, the second gun shoots so inaccurately that it seriously impairs the shooting of the battery even after calibration; and its inaccuracy should be remedied or the gun or carriage replaced.

4. Find the percentage of hits to be expected of the second gun alone.

NOTE: When a gun is calibrated by changing the relative positions of pointer and range-scales it is assumed that the carriage has an angular jump (positive or negative) that is constant for all ranges. This is the best that can be done, and is practically satisfactory.

Thus the jump in gun No. 1 is negative and equivalent to 75 yards at that range. But at 6300, $11' = 200$ yards. Hence the assumed jump is $-\frac{75}{200} \times 11' = -4'.1$.

The term jump as here used may include not only the action of the carriage but also virtual jump, "kiting" of projectile, slight unsteadiness in flight changing inclination of trajectory, etc.

Tables A, B, C, and D are, respectively, a table of values of δ_1/δ, the atmosphere factor; a table of percentage changes in V due to change in temperature of powder; a range-table for the 10-inch B. L. Rifle; and a table of data for the construction of range-correcting devices. Table A is reprinted from Artillery Note No. 25; Table B and the formula given are deduced from results obtained by the Ordnance Department at Sandy Hook.

APPENDIX I.

I. THE ATMOSPHERIC DENSITY FACTOR $\frac{\delta_{\prime}}{\delta}$

This ratio is necessary in Ballistics for two purposes, viz.;

(1) In order to reduce the results of experimental firings to standard atmospheric conditions.

(2) In order to find from a given range-table, computed for standard conditions, what range to expect for a given set of conditions not standard.

The standard conditions referred to are:

Barometer, 750 mm. = 29.5275 inches.
Thermometer, 15° C, = 59° Fahr.
Relative humidity, 50 per cent., or ½.

The formula will first be deduced for a zero humidity, and then modified to conform to a relative humidity represented by *k*, which is always a proper fraction, since the relative humidity cannot exceed 100 per cent.

Dry air is suppósed to behave as a perfect gas; and, therefore, to comform to the law

$$pv = p_0 v_0 \cdot \frac{460 + t}{460 + 32} = p_0 v_0 \cdot \frac{460 + t}{492} \qquad (1)$$

in which t is the temperature Fahr., and v and p the volume and pressure of a given mass of gas at that temperature. p_0 and v_0 are any assumed pressure and its corresponding volume, for this same mass of gas, at a temperature 32° Fahr.

Now, since, for a given mass of gas, the volume and density are inversely proportional, we have, from (1),

$$\frac{\delta_0}{\delta} = \frac{v}{v_0} = \frac{p_0}{p} \cdot \frac{460 + t}{492} \qquad (2)$$

in which δ_0 is the density of the given mass, under pressure p_0 and at a tempature 32° Fahr.; and δ the density of the same mass, under pressure p, and at temperature t° Fahr.

Now the atmosphere is composed, not dry air alone, but of dry air and water-vapor mixed; and experience has shown that while each of these exerts pressure, so that the total pressure is the sum of the two separate pressures; that is,

$$p = p_a + p_v \qquad (3)$$

at the same time, the densities of the two are determined rather by the common barometric pressure than according to the law of perfect gases. Consequently, since, under identical conditions of temperature and pressure, the density of water-vapor is only $\frac{5}{8}$ as great as that of dry air, it follows that the pressure p_v represents only $\frac{5}{8}$ as much mass as if it had been due to dry air; and this affords a simple means of returning to the formula for the density of dry air, (Formula 2); and, by introducing a suitable correction of the recorded barometric reading, p, based on the actual conditions of humidity, a formula applicable to all conditions met with in practice may be produced.

For the purpose of obtaining the vapor-pressure, the vapor-pressure in saturated space has been recorded by Regnault; and such vapor-pressures, known as saturation vapor-pressures, were measured at each temperature used in the experiments, the water being placed in a vacuum and allowed to evaporate until the pressure was a maximum for that temperature.

The results of the experiments were afterwards adjusted by Dr. Broch, who used an empirical formula, determining his constants from observation-equations, by least squares.

The following table was abridged from one calculated from his formula and published in the "Smithsonian Meteorological Tables":

SATURATION VAPOR-PRESSURE, F

VALUE OF F IN INCHES OF BAROMETRIC PRESSUTE

(Temperatures on Fahrenheit scale)

t°	0	1	2	3	4	5	6	7	8	9
0	0.05	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07
10	0.07	0.07	0.08	0.08	0.08	0.09	0.09	0.10	0.10	0.11
20	0.11	0.11	0.12	0.12	0.13	0.14	0.14	0.15	0.15	0.16
30	0.17	0.17	0.18	0.19	0.19	0.20	0.21	0.22	0.23	0.24
40	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.32	0.33	0.35
50	0.36	0.37	0.39	0.40	0.42	0.43	0.45	0.46	0.48	0.50
60	0.52	0.54	0.56	0.58	0.60	0.62	0.64	0.66	0.68	0.71
70	0.73	0.76	0.78	0.81	0.84	0.87	0.90	0.93	0.96	0.99
80	1.02	1.06	1.09	1.13	1.16	1.20	1.24	1.28	1.32	1.36
90	1.41	1.45	1.50	1.55	1.60	1.65	1.70	1.75	1.80	1.86
100	1.92	1.97	2.03	2.10	2.16	2.23	2.29	2.36	2.43	2.50

If water-vapor and air, when intermixed, as they are in atmospheric air, behaved as perfect gases, we would have the above vapor pressures at the seacoast, as the space above the water would be saturated with vapor, since the water is in contact with it.

But saturation seldom occurs, the percentage of saturation varying, for practical purposes, between 30 per cent, and 90 per cent.

For a general formula, then, we may use the fraction *k*, and then, assume for particular classes of localities, a mean value of *k*, provided the variation of humidity in any proposed class of localities is not too great.

Using the general value, *k* of the humidity, we may correct the reading of the barometer so that it may be used in the formula for density, as follows:

Let p′ be the reading that would, if produced by dry air, pertain to the actual density for which a barometric reading p is recorded; then, p_v being the vapor-pressure at the time,

$$p' = p_a + \tfrac{5}{8}\,p_v = p - \tfrac{3}{8}\,p_v = p - \tfrac{3}{8}\,kF \qquad (4)$$

For a barometric pressure p′, for dry air, we have from (2)

$$\frac{\delta_0}{\delta} = \frac{p_0}{p'} \cdot \frac{460 + t}{492} \qquad (5)$$

Using with this formula the values of the readings for standard conditions; viz: bar: 29.5275; ther. 59°; $k = \frac{1}{2}$, we have representing the standard density by $\delta_{,}$

$$\frac{\delta_0}{\delta_{,}} = \frac{p_0}{29.4335} \cdot \frac{519}{492} \qquad (6)$$

since F = 0.50, for standard conditions.

Dividing (5) by (6), member by member, we have

$$\frac{\delta_{,}}{\delta} = \Big[8.75367 - 10\Big] \cdot \frac{460 + t}{p - \frac{3}{8}\,kF} \qquad (7)$$

From this formula, we might compute a table of values of $\frac{\delta_{,}}{\delta}$ for any assumed mean percentage of saturation; e. g., for $k = 0$ that is, for dry air; or for $k = \frac{1}{2}$, i. e. 50% saturation, or as in Artillery Notes No. 25, for 78% r. h.

Taking k as $\frac{4}{5}$, which approximates sufficiently to the latter for all practical purposes, our formula becomes

$$\frac{\delta_{,}}{\delta} = \Big[8.75367 - 10\Big] \cdot \frac{460 + t}{p - 0.3\,F} \qquad (7a)$$

EXAMPLES OF THE APPLICATION OF FORMULAS (7) AND (7a)

1. Compute $\frac{\delta_{,}}{\delta}$ for 60° and barometer 29 inches; 80 per cent relative humidity.

Analysis:

$$F = 0.52 \text{ and } 0.3\,F = .156$$
$$p' = 29 - .156 = 28.844$$
$$460 + t = 460 + 60 = 520$$

Computation:

$$\text{Const. log} = 8.75367-10$$
$$\log 520 = 2.71600$$
$$\text{a. c. log } 28{\cdot}844 = \underline{8.54000-10}$$

$$\log \frac{\delta_{\prime}}{\delta} = 0.00967$$

$$\frac{\delta_{\prime}}{\delta} = 1.023$$

2. Compute $\frac{\delta_{\prime}}{\delta}$ for temperature 80°; barometer 30 in, and 50 per cent relative humidity.

Analysis:

$$F = 1.02 \qquad k = \tfrac{1}{2}$$
$$p' = 30 - .191 = 29.809$$
$$460 + t = 460 + 80 = 540$$

Computation:

$$\text{Const. log} = 8.75367-10$$
$$\log 540 = 2.73239$$
$$\text{a. c. log } 29.809 = \underline{8.52565-10}$$

$$\log \frac{\delta_{\prime}}{\delta} = 0.01171$$

$$\frac{\delta_{\prime}}{\delta} = 1.027$$

3. Find for relative humidity 80% the values of $\frac{\delta_{\prime}}{\delta}$ to 4 places of decimals for each of the barometric heights 28, 29, 30, 31 inches; and for each 10° from 0° to 100°, inclusive. Then interpolate to single degrees of temperature by first differences and *then* reduce result to three places of decimals. Check results by comparison with $\frac{\delta_{\prime}}{\delta}$ table in Artillery Notes No. 25, which is the table used throughout the course.

4. Given for a particular shot:

barometer.................................30″.178
thermometer..........................84°
humidity .... .40

Find $\frac{\delta_{\prime}}{\delta}$.

Analysis:

$$F = 1.16 : k = .40 : \tfrac{3}{8} kF = .174$$

$$p' = 30.178 - .174$$

$$= 30.004$$

$$460 + t = 460 + 84 = 544$$

Computation:

$$\text{Const. log.} = 8.75367 - 10$$

$$\log 544 = 2.73560$$

$$\text{a. c. } \log p' = 8.52282 - 10$$

$$\log \frac{\delta_{\prime}}{\delta} = 0.01209$$

$$\frac{\delta_{\prime}}{\delta} = 1.028$$

II. THE ALTITUDE FACTOR f_a.

Find by calculation from formula 7a the following table of values of $\frac{\delta_{\prime}}{\delta}$.

t	28"	29"	30"	31"
0°	.932	.900	.870	.842
10°	.953	.920	.889	.860
20°	.973	.940	.908	.879
30°	.994	.960	.928	.898
40°	1.015	.980	.948	.917
50°	1.037	1.001	.968	.936
60°	1.059	1.022	.988	.956
70°	1.082	1.044	1.009	.977
80°	1.106	1.067	1.031	.998
90°	1.131	1.092	1.055	1.020
100°	1.158	1.117	1.079	1.044

It may be shown from the data in the above table that, in the average,

1" barometer corresponds to .0337 of $\frac{\delta_{\prime}}{\delta}$

An examination of Table No. 3, page 53 (Reduction of barometer readings to sea-level) Artillery Circular D shows that a change in height of 1000 feet, the temperature remaining the same, corresponds on an average of 1.1 inches. The temperature falls with increase of height at the rate of 1° per 300 feet. Hence $\Delta t = -3\tfrac{1}{3}°$ per 1000 feet rise. This, with the above table affords a means of calculating the altitude factor $f_a = \frac{\delta}{\delta_h}$ (See Part I., page 11.)

We have per 1000 feet of height, for

Drop of 1".1 in barometer	$+.037$ of $\frac{\delta_1}{\delta}$
Drop of 3⅓° in temperature	$-.007$ of $\frac{\delta_1}{\delta}$
The effect of these causes is	$+\ .03$ of $\frac{\delta_1}{\delta}$

Hence the altitude factor fa is

$$1 + .03 \cdot \frac{h}{1000}$$

and since in direct fire h is taken as ⅔ y_0 we have

$$f_a = 1 + .00002\, y_0 = 1 + .02\left(\frac{y_0}{1000}\right)$$

For great heights a different formula would be employed, as the data as to reduction of barometer to sea level was used for 1000 feet, and for great heights the rate of change here used is only approximately true.

APPENDIX II.

THE WIND FACTOR, f_w.

In dealing with the effect of wind on the range, we use W_x to represent the component of the wind in miles per hour in the plane of fire. Representing this wind component in feet per second by W, we have $W = \frac{44}{30} W_x$.

The formula for the retardation of unit projectile is

$$Cr = Av^n$$

and the mean value of n in the trajectory is, for our service muzzle velocities, 2.

Hence, as a basis, we take

$$Cr = Av^2.$$

For a horizontal retardation r_x we have

$$Cr_x = Av^2_x$$

The mean value of v_x the horizontal velocity in the trajectory is

$$v_x = \frac{X}{T}$$

With the wind blowing W miles per hour, *with* the projectile the horizontal velocity relatively to the moving air would have for its mean value

$$\frac{X}{T} - W$$

Hence the actual horizontal retardation becomes under these conditions,

$$r_w = \frac{A\left(\frac{X}{T} - W\right)^2}{C}$$

If, now, we suppose that the retardation was due to a changed C instead of an increased relative velocity, we produce the same effect as follows

$$r_w = \frac{\left(\frac{X}{T} - W\right)^2}{C} = \frac{A\left(\frac{X}{T}\right)^2}{C_w}$$

we find

$$\frac{C_w}{C} = f_w = 1 \div \left(1 - \frac{TW}{X}\right)^2 = 1 + \frac{2TW}{X} + \text{etc.}$$

Hence $f_w = 1 + \frac{2WT}{X}$

Now the wind velocity increases with the height above the surface of the earth. The rate of increase is such that at about 1000 feet the wind has twice the speed at the surface and at 25000 feet about three times that speed.

To find the power of the height with which the speed varies we have

$$(25)^n = \tfrac{3}{2}$$

From this it will be found that $n = \frac{1}{8}$, nearly. The mean height in the trajectory is $\frac{2}{3} y_0$ and $y_0 = \frac{1}{8} gT^2$, nearly.

Hence, the wind velocity varies as the eighth root of the maximum ordinate and hence as the fourth root of the time of flight.

The wind velocity is found from the anemometer in seacoast work and the mean height of this instrument should be about 60 feet. Taking 66 feet as its value, the anemometer wind velocity may be taken as that of a trajectory having a maximum ordinate of 100 feet or a time of flight 5 seconds. Hence the anemometer wind velocity should be multiplied by $\sqrt[4]{\frac{T}{5}}$, for use in the formula for the wind factor.

Further, the anemometer readings are in miles per hour; hence we must introduce W_x into the formula, as follows:

$$f_w = 1 + \frac{2W_x T}{X} \times \frac{44}{30} \times \sqrt[4]{\frac{T}{5}} = 1 + \frac{2W_x T^{\frac{5}{4}}}{X}$$

W_x being positive if a rear wind and negative if a head wind, that is to say

$$f_w = 1 \pm \frac{2W_x T^{\frac{5}{4}}}{X}$$

the upper sign being used for a rear and the lower for a head-wind component.*

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* For firings through screens to determine the coefficient of form the wind velocity is best taken close to the surface; and in any case the formula for such a low trajectory would be without the factor $\sqrt[4]{\frac{T}{5}}$

Hence we would have, for that case only,

$$f_w = 1 \pm \frac{3W_x T}{X}$$

APPENDIX III.

INTERPOLATION BY DIFFERENCES

Let a quantity vary in accordance with a law, which law need not be known except in a general way from a set of values of the quantity corresponding to a set of values of another quantity taken at regular intervals; for example, suppose we have given:

R	T
1000	1.37
4000	5.85
7000	11.11
10000	17.36
13000	25.17

We have values of R at regular intervals, and corresponding values of T. It is required from the values furnished to interpolate intermediate values which shall follow a law consistent with that of the given values.

To do this we may resort to the method of differences as follows:

Given the quantities a, b, c, d, e, etc., as the values corresponding to values of a related quantity taken at regular intervals, to find any intermediate value corresponding to an intermediate value of the related quantity, called the argument.

1st. Subtract each value from the next following and designate the remainder δ_1.

2nd. Arrange these differences in order and treat them in the same way. Designate their remainders δ_2.

3rd. Do the same with the new set of differences and obtain δ_3, etc.

4th. Continue until the differences are small, or zero, or the terms given afford no more differences.

To illustrate:

R	T	δ_1	δ_2	δ_3	δ_4
1000	1.37	4.48	.78	.21	.36
4000	5.85	5.26	.99	.57	
7000	11.11	6.25	1.56		
10000	16.36	7.81			
13000	25.17				

Here $a = 1.37$; $b = 5.85$; $c = 11.11$; $d = 16.36$; $e = 25.17$; $b - a = 4.48$; $c - b = 5.26$; $d - c = 6.25$; $e - d = 7.81$.

These latter are tabulated as δ_1 and their differences (tabulated as δ_2) are:

$$c - b - (b - a) = c - 2b + a = 0.75$$
$$d - c - (c - b) = d - 2c + b = 0.99$$
$$e - d - (d - c) = e - 2d + c = 1.56$$

The δ_3 column is obtained in the same way from the δ_2 column, and so on.

Considering now only the first horizontal line of the above table, we write:

$$a = 1.37;\ \delta_1 = 4.48;\ \delta_2 = 0.78;\ \delta_3 = 0.21;\ \delta_4 = 0.36$$

and this is all that is needed for interpolating.

For, reverting to the letters, we find

$$\begin{aligned} b - a &= \delta_1 & \therefore\ \ b &= a + \delta_1 \\ c - 2b + a &= \delta_2 & \therefore\ \ c &= \delta_2 + 2b - a = a + 2\delta_1 + \delta_2 \\ d - 3c + 3b - a &= \delta_3 & \therefore\ \ d &= a + 3\delta_1 + 3\delta_2 + \delta_3 \\ e - 4d + 6c - 4b + a &= \delta_4 & \therefore\ \ e &= a + 4\delta_1 + 6\delta_2 + 5\delta_3 + \delta_4 \end{aligned}$$

and, in general, for a term the nth after the first

$$T_n = a + \frac{n\,\delta_1}{1} + \frac{n\,(n-1)}{1.2}\delta_2 + \frac{n\,(n-1)\,(n-2)}{1.2.3}\delta_3$$

$$+ \cdots\cdots + \frac{\underline{|n}}{\underline{|n-m}\ \underline{|m}}\,\delta_m + \text{etc.}$$

To illustrate the use of this formula, let us take the numerical example used.

The term corresponding to R = 1000 is a
" " " " R = 4000 is the 1st after a $\therefore$ n = 1
" " " " R = 7000 is the 2d after a $\therefore$ n = 2
" " " " R = 10000 is the 3d after a $\therefore$ n = 3

and so on.

Suppose it is required to find the values of T for every thousand of R; we have,

R	n	R	n
2000	$\frac{1}{3}$	7000	2
3000	$\frac{2}{3}$	8000	$\frac{7}{3}$
4000	1	9000	$\frac{8}{3}$
5000	$\frac{4}{3}$	10000	3
6000	$\frac{5}{3}$	11000	$\frac{10}{3}$

Hence, for 2000,

$$T = 1.37 + \tfrac{1}{3}(4.48) + \tfrac{1}{3} \times -\tfrac{2}{3} \times \tfrac{1}{2} \times .78 + \tfrac{1}{3} \times -\tfrac{2}{3} \times -\tfrac{5}{3} \times \tfrac{1}{6} \times .21$$
$$+ \tfrac{1}{3} \times -\tfrac{2}{3} \times -\tfrac{5}{3} \times -\tfrac{8}{3} \times \tfrac{1}{24} \times .36$$
$$= 1.37 + 1.493 - .087 + .013 - .015$$
$$= 1.37 + 1.493 - .087 + .013 - .015 = 2.774$$

For 3000

$$\begin{aligned} T &= 1.37 + \tfrac{2}{3}(4.48) + \tfrac{2}{3} \times -\tfrac{1}{3} \times \tfrac{1}{2} \times .78 + \tfrac{2}{3} \times -\tfrac{1}{3} \\ &\quad \times -\tfrac{4}{3} \times \tfrac{1}{6} \times .21 + \tfrac{2}{3} \times -\tfrac{1}{3} \times -\tfrac{4}{3} \times -\tfrac{7}{3} \times \tfrac{1}{24} \times .36 \\ &= 1.37 + 2.987 - .087 + .010 - .010 \\ &= 4.27. \end{aligned}$$

We may continue thus, making $n = \frac{5}{3}, \frac{7}{3}, \frac{8}{3}, \frac{10}{3}$ and obtain the values of T for the missing ranges without difficulty.

The values taken from the range table are

R	T	R	T
2000	2.78	8000	13.06
3000	4.27	9000	15.14
5000	7.52	11000	19.75
6000	9.27	12000	22.35

Let the student obtain the values for these ranges by interpolation and compare.

It is frequently necessary to interpolate values from ranges 1000 to 3000 yards apart, and as a rule second differences are regarded as sufficient.

In the following methods of interpolating any number of means between values, all differences of a higher order than the second will be neglected.

Our formula becomes

$$T_n = a + n\,\delta_1 + \frac{n(n-1)\delta_2}{2}$$

Suppose we desire the interval between successive values to be only $1/m$ of what it was; then making $n = 1/m, 2/m, 3/m,$ ....$(m-1)/m$ we get the intermediate terms.

The terms are in order :

$$a + \frac{\delta_1}{m} + \frac{1}{2m}\left(\frac{1}{m} - 1\right)\delta_2 = a + \frac{\delta_1}{m} - \frac{(m-1)}{2m^2}\delta_2$$

$$a + \frac{2\delta_1}{m} + \frac{2}{2m}\left(\frac{2}{m} - 1\right)\delta_2 = a + \frac{2\delta_1}{m} - \frac{2(m-2)}{2m^2}\delta_2$$

$$a + \frac{3\delta_1}{m} + \frac{3}{2m}\left(\frac{3}{m} - 1\right)\delta_2 = a + \frac{3\delta_1}{m} - \frac{3(m-3)}{2m^2}\delta_2$$

If we now take the differences between these successive values and call them $\varDelta_1, \varDelta_2, \varDelta_3,$ ....$\varDelta_m$ we have :

$$\varDelta_1 = \frac{\delta_1}{m} - \frac{m-1}{2m^2}\delta_2$$

$$\varDelta_2 = \frac{\delta_1}{m} - \frac{m-3}{2m^2}\delta_2$$

$$\varDelta_3 = \frac{\delta_1}{m} - \frac{m-5}{2m^2}\delta_2$$

etc. etc.

Placing now m = 3, 4,............................... 10 we have

m = 3

$$\varDelta_1 = \frac{\delta_1}{3} - \tfrac{1}{9}\,\delta_2$$

$$\varDelta_2 = \frac{\delta_1}{3}$$

$$\varDelta_4 = \frac{\delta_1}{3} + \tfrac{1}{9}\,\delta_3$$

m = 4

$$\varDelta_1 = \frac{\delta_1}{4} - \tfrac{3}{32}\,\delta_2$$

$$\varDelta_2 = \frac{\delta_1}{4} - \tfrac{1}{32}\,\delta_2$$

$$\varDelta_3 = \frac{\delta_1}{4} + \tfrac{1}{32}\,\delta_2$$

$$\varDelta_4 = \frac{\delta_1}{4} + \tfrac{3}{32}\,\delta_3$$

m = 5

$$\varDelta_1 = \frac{\delta_1}{5} - \tfrac{2}{25}\,\delta_2$$

$$\varDelta_2 = \frac{\delta_1}{5} - \tfrac{1}{25}\,\delta_2$$

$$\varDelta_3 = \frac{\delta_1}{5}$$

$$\varDelta_4 = \frac{\delta_4}{5} + \tfrac{1}{25}\,\delta_2$$

$$\varDelta_5 = \frac{\delta_1}{5} + \tfrac{2}{25}\,\delta_2$$

m = 10

$$\varDelta_1 = \frac{\delta_1}{10} - \tfrac{9}{200}\,\delta_2$$

$$\varDelta_3 = \frac{\delta_1}{10} - \tfrac{7}{200}\,\delta_2$$

$$\varDelta_3 = \frac{\delta_1}{10} - \tfrac{5}{260}\,\delta_2$$

$$\varDelta_4 = \frac{\delta_1}{10} - \tfrac{3}{200}\,\delta_2$$

$$\varDelta_5 = \frac{\delta_1}{10} - \tfrac{1}{200}\,\delta_2$$

$$\varDelta_6 = \frac{\delta_1}{10} + \tfrac{1}{200}\,\delta_2$$

$$\varDelta_7 = \frac{\delta_1}{10} + \tfrac{3}{200}\,\delta_3$$

$$\varDelta_8 = \frac{\delta_1}{10} + \tfrac{5}{200}\,\delta_2$$

$$\varDelta_9 = \frac{\delta_1}{10} + \tfrac{7}{200}\,\delta_2$$

$$\varDelta_{10} = \frac{\delta_1}{10} + \tfrac{9}{200}\,\delta_2$$

Representing $\frac{\delta_1}{m}$ by A, and $+\frac{\delta_2}{2\,m^2}$ by B, we have, generally, whatever the value of m,

$$\varDelta_1 = A - (m - 1)\,B$$
$$\varDelta_2 = A - (m - 3)\,B = \varDelta_1 + 2B$$
$$\varDelta_3 = A - (m - 5)\,B = \varDelta_2 + 2B$$
$$\varDelta_4 = A - (m - 7)\,B = \varDelta_3 + 2B$$
$$\varDelta_5 = A - (m - 9)\,B = \varDelta_4 + 2B$$

etc., etc., ad libitum, according to the value of m.

For m = 3; $A = \delta_1/3$ and $B = +\,\delta_2/18$

For m = 4; $A = \delta_1/4$ and $B = +\,\delta_2/32$

For $m = 5$; $A = \delta_1/5$ and $B = + \delta_2/50$
For $m = 10$; $A = \delta_1/10$ and $B = + \delta_2/200$

Suppose now values are interpolated in the numerical example with $m = 3$. We have, using second differences only,

$$A = \tfrac{1}{3} \cdot \delta_1 , \quad B = \delta_2/18.$$

Hence for 2000

$$A = \frac{4.48}{3} = 1.493 ; \qquad B = \frac{.78}{18} = .04333$$

and

$$\begin{aligned} \Delta_1 &= A - (m - 1) B = A - 2B = 1.493 - .087 = 1.406 \\ \Delta_2 &= \Delta_1 + 2B \qquad = 1.406 + .087 = 1.493 \\ \Delta_3 &= \Delta_2 + 2B \qquad = 1.493 + .087 = 1.580 \end{aligned}$$

Hence for

$$\begin{aligned} R &= 2000, \ T = 1.37 + \Delta_1 = 2.776 = 2.78 \\ R &= 3000, \ T = 2.776 + \Delta_2 = 4.269 = 4.27 \\ R &= 4000, \ T = 4.269 + \Delta_3 = 5.849 = 5.85 \end{aligned}$$

Now, owing to the fact that third differences are neglected and that the cummulative effect will appear, we begin again at 4000 yards using

$$a = 5.85 ; \ \delta_1 = 5.26 : \ \delta_2 = .99 ; \text{ and hence}$$

$$\begin{aligned} &A = 1.753 ; \quad B = .055 ; \quad 2B = 0.110 \\ &\Delta_1 = A - 2B = 1.753 - .110 = 1.643 \\ &\Delta_2 = \Delta_1 + 2B \qquad = 1.753 \\ &\Delta_3 = \Delta_2 + 2B \qquad = 1.863 \end{aligned}$$

Hence for

$$\begin{aligned} R = 5000, \ T &= 5.850 + 1.643 = 7.493 \\ 6000, \ T &= 7.493 + 1.753 = 9.246 \\ 7000, \ T &= 9.346 + 1.863 = 11.109 \end{aligned}$$

The case where $m = 10$ is an important one, and it will be desirable at the same time to illustrate a case where the successive differences do not increase in order or may be negative—

R	v_ω	δ_1	δ_2
1000	2153	—269	+10
4000	1884	—259	+28
7000	1623	—231	+44
10000	1392	—187	
13000	1205		

Find v_ω for each 300 yards between 7000 and 10000 yards.

Here $\delta_1 = - 231$; $\delta_2 = + 44$

$A = \delta_1/10 = - 23.1$; $B_1 = \delta_2/200 = .22$ $\quad 2B = .44$

Hence

$$\begin{aligned}
\Delta_1 &= A - 9B = -23.10 - 1.98 = -25.08 \\
\Delta_2 &= \Delta_1 + 2B = -24.64 \\
\Delta_3 &= -24.20 \\
\Delta_4 &= -23.76 \\
\Delta_5 &= -23.32 \\
\Delta_6 &= -22.88 \\
\Delta_7 &= -22.44 \\
\Delta_8 &= -22.00 \\
\Delta_9 &= -21.56 \\
\Delta_{10} &= -21.12
\end{aligned}$$

Hence

R	v_ω	Δ	Range table v_ω
7000	1623.00	—25.08	1623
7300	1597.92	—24.64	
7600	1573.28	—24.20	1574
7900	1549.08	—23.76	
8200	1525.32	—23.32	1526
8500	1502.00	—22.88	
8800	1479.12	—22.44	1480
9100	1456.68	—22.00	
9400	1434.68	—21.56	1435
9700	1413.12	—21.12	
10000	1392.00		1392

It will be noticed that there is in no case a difference of 1 foot per second.

THE USE OF DIFFERENCES IN DETECTING ERRORS OF CALCULATION OR OBSERVATION.

Where it is known that a quantity varies very gradually and regularly, it is safe to assume the third order of differences constant. On the other hand, if a set of observed or calculated values are somewhat irregular, imperfections may be eliminated by assuming δ_3, uniform.

To show the use of differences in correcting such errors the following given in the Rev. Francis Bashforth's "Motion of Projectiles" is of value :

'The following example is given as illustrating the great utility of differencing when systematic experiments or independent calculations are being made.

Suppose the following logarithms of numbers had been independently calculated, and that the calculator wished to test the accuracy of his work.

Numbers	Calculated logarithms	δ_1	δ_2	Corrections
1280	.107210	+3380	—27	
1290	.110590	3353	35	+10
1300	.113943	3318	05	—20
1310	.117261	3313	35	+10
1320	.120574	3278	24	— 1
1330	.123852	3254	26	+ 2
1340	.127106	3228	23	— 1
1350	.130334	3205	23	
1360	.133539	3182		
1370	.136721			

Applying these corrections to the column of δ_2 values we get a new set of values of δ_2, giving a sensibly constant δ_3; using the new values of δ_2, we construct new values of δ_1, and thence of the logarithms.

Numbers	Corrected log	δ_1	δ_2
1280	.107210	+3380	—27
1290	.110590	3353	—25
1300	.113943	3328	—25
1310	.117271	3303	—25
1320	.120574	3278	—25
1330	.123852	3253	—24
1340	.127105	3229	—24
1350	.130334	3205	—23
1360	.133539	3182	
1370	.136721		

The difference of type indicates the erroneous figures in the first table and their corrections in the other.'

A method similar to this is in use in our service in checking calculations in range tables. It is also used in calculations of corrections of elevation due to curvature and height of site above mean low water. After the calculations are completed they are checked by making the second or third differences practically constant according to the interval of calculation.

APPLICATIONS.

1. THE USE OF DIFFERENCES IN CALCULATING RANGE TABLES.

Much of the labor of computing range tables may be avoided by the use of differences.

In the first place, it is possible by calculating log C as is done on pp. 17 and 18 of Part I, for zero, 5000 and 10000 yards range, and taking these, for purposes of interpolation, to five places, we may calculate by differences a closely approximate value of log C for every thousand yards range.

The value of C for zero range is $C_0 = \frac{2\,w^2}{cd^5}$, since $f_a = 1$ and $\varphi + \omega = 0$.

Now $\gamma = \frac{d^3}{2w}\left\{1 + \frac{\varphi + \omega}{135}\right\}$. Representing the factor in parenthesis by γ' we have

$$\gamma' = 1 + \frac{\varphi + \omega}{135}$$

and

$$C = f_a \cdot \frac{2\,w^2}{c\gamma'\,d^5}.$$

$$C_0 = \frac{2\,w^2}{c\gamma'\,d^5}$$

Hence

$$C = \frac{f_a}{\gamma'} \cdot C_0$$

Now f_a and γ' may be much more briefly calculated if we have a closely approximate value of the true log C. This we may find by differences as above indicated.

An illustration will be given using the 8-inch, 10-inch and 12-inch guns, the weights of projectiles being 316, 604 and 1046 pounds, and the muzzle velocities 2200, 2250 and 2250, respectively.

We have for log C_0 in the three cases 0.78484 ; 0.86309 ; and 0.94419, respectively. Calculations for 5000 give for log C in the three cases 0.76022 ; 0.84247 ; 0.92455 ; and, for 10000 yards, 0.72189 ; 0.81190 ; 0.89834.

The computations may be arranged as follows :

	8″	10″	12″
R	log C	log C	log C
0	.78484	.86309	.94419
5000	.76022	.84247	.92455
1000	.72189	.81190	.89834
δ_1	—.02462	—.02062	—.01964
δ_2	—.01371	—.00995	—.00657
A	—.004924	—.004124	—.003928
B	—.0002742	—.0001990	—.0001314
A — 4B	—.003827	—.003328	—.003402+
2B	—.000548+	—.000398	—.000263—

	8″	10″	12″
Δ_1	—.003827	—.003348	—.003402
Δ_2	—.004375	—.003726	—.003665
Δ_3	—.004924	—.004124	—.003928
Δ_4	—.005472	—.004522	—.004191
Δ_5	—.006020	—.004920	—.004454
Δ_6	—.006569	—.005318	—.004716
Δ_7	—.007117	—.005716	—.004979
Δ_8	—.007666	—.006114	—.005242
Δ_9	—.008214	—.006512	—.005505
Δ_{10}	—.008763	—.006910	—.005768
Δ_{11}	—.009311	—.007308	—.006030
Δ_{12}	—.009859	—.007706	—.006293
Δ_{13}	—.010408	—.008104	—.006556

Applying these differences we find the following interpolated values of log C for ranges at intervals of 1000 yards and beside them are placed the computed values from the range tables showing how closely these values approximate to those of the true log C and hence that they will answer admirably for computing γ' and f_a.

Values of log C.

	8-inch		10-inch		12-inch	
R	Int.	True	Int.	True	Int.	True
0000	.78484	.78484	.86309	.86309	.94419	.94419
1000	.78101	.78118	.85976	.85922	.94079	.94034
2000	.77664	.77602	.85604	.85574	.93712	.93690
3000	.77171	.77081	.85191	.85177	.93320	.93310
4000	.76624	.76555	.84739	.84733	.92900	.92897
5000	.76022	.76022	.84247	.84247	.92455	.92455
6000	.75365	.75359	.83715	.83724	.91983	.91986
7000	.74654	.74620	.83144	.83167	.91486	.91492
8000	.73887	.73823	.82532	.82572	.90961	.90971
9000	.73065	.73006	.81881	.81922	.90411	.90420
10000	.72189	.72189	.81190	.81190	.89834	.89834
11000	.71258	.71371	.80459	.80345	.89231	.89208
12000	.70272		.79689	.79351	.88602	.88536
13000	.69222		.78878		.87946	.87812

It will be noted that the greatest percentage error in the trial C is that for the 10-inch gun at 12000 yards. The difference of the logarithms is then .00338. The ratio of the interpolated value to the true value is in this extreme case 1.008:1, or the difference between the two C's is less than 1%.

As an illustration of the use to which these interpolated values may be put, the value of C will be found for the 12-inch gun for 9000 yards range using the interpolated C as a basis of computation.

$$\begin{aligned} \log X &= 4.43136 \\ \text{a.c. } \log C &= 9.09589 \\ \hline \log Z &= 3.52725 \\ Z &= 3367 \end{aligned}$$

As on page ~~18~~ 20, Part I, we find,

$$1 + \frac{\varphi + \omega}{135} = 1.1166, \text{ and } f_a = 1.0184$$

Now

$$C = \frac{f_a}{\gamma'} . C_0$$

$$\begin{aligned} \log f_a &= .00792 \\ \text{a.c. } \log \gamma' &= 9.95210 \\ \log C_0 &= .94419 \\ \hline \log C &= 0.90421 \end{aligned}$$

APPLICATION II.

Second differences are sensibly constant in relation to range-corrections in the vicinity of a given range.

The necessity for plotting each range-correction opposite a non-tabular corrected range (as in Part I), is obviated by the method about to be deduced for finding the correction by the use of a tabular corrected range, opposite which this corrected range is plotted.

The difference in the two procedures may be illustrated thus: With 9000 yards as the uncorrected range we found for changes in C of +20%, +10%, —10% and —20% the following values ΔR: +366, +200, —259 and —591, respectively, and these were to be plotted opposite the corrected ranges 8634, 8800, 9259 and 9591, respectively.

Using the data for the other ranges we determine similar groups for each of them. Finally, curves are drawn with each correction opposite the corrected range and we might then scale off the values of ΔR opposite 9000 yards. We should find thus that for a corrected range 9000 yards we would have in round numbers +400, +210, —245 and —530 yards, respectively. It is desired to find these directly using the corrected range (9000 yards in the case quoted) as a basis of computation.

If second differences are constant, we have

$$\Delta R = h + kR + l\,R^2$$

since there could be no term in R^3.

Of the three constants h, k and l, h is zero for all corrections due to changes in V, C and W_x; k is zero for all changes in C and W_x; and k is $2\frac{\Delta V}{V}$ for changes in V; l is negative for velocities and positive for C and W_x.

Taking the general expression and representing the corrected range by $R' = R - \Delta R$ we have

$$\frac{\Delta R}{R} = k + l\,R$$

$$\frac{\Delta R'}{R'} = k + l\,R'$$

whence, by division,

$$\frac{\Delta R}{\Delta R'} = \frac{R}{R'} \cdot \frac{k + l\,R}{k + l\,R'} = \left(1 + \frac{\Delta R}{R'}\right)\left(1 + l\,R'\,\frac{\Delta R}{\Delta R'}\right)$$

$$= \left(1 + \frac{\Delta R'}{R'} \cdot \frac{\Delta R}{\Delta R'}\right)\left(1 + \left(\frac{\Delta R'}{R'} - k\right)\frac{\Delta R}{\Delta R'}\right)$$

For k = 0 we find

$$\frac{\Delta R}{\Delta R'} = 1 + \frac{2\,\Delta R'}{R'} + 5\left(\frac{\Delta R'}{R'}\right)^2 + \text{etc.}$$

or

$$\frac{\Delta R}{R'} = \frac{\Delta R'}{R'}\left\{1 + 2\frac{\Delta R'}{R'} + 5\left(\frac{\Delta R'}{R'}\right)^2\right\}$$

The values of $\frac{\Delta R}{R'}$ are tabulated for use in C and W_x changes, with $\frac{\Delta R'}{R'}$ as argument.

Table I is for positive values of $\Delta R'$.

Table II is for negative values of $\Delta R'$.

For $k = 2\frac{\Delta V}{V}$ we find after reduction and neglect of insignificant terms, as before,

$$\frac{\Delta R}{\Delta R'} = 1 + 2\left(\frac{\Delta R'}{R'} - \frac{\Delta V}{V}\right).$$

The use of these formulas will be illustrated by the following typical case.

Given values of $\Delta R'$, for 9000 yards, for 12″ gun $v = 2250$, the values of $\Delta R'$ corresponding to the changes in C, W_x and V indicated, find ΔR in each case, ($R' = 9000$).

Solution:

I	$\Delta C/C$	$\Delta R'$	$\Delta R'/R'$	$\Delta R/R'$	From	ΔR
	+.20	+366	.0407	.0443	Table I	+399
	+.10	+200	.0222	.0232		+209
	—.10	—259	—.0288	—.0272	Table II	—245
	—.20	—591	—.0657	—.0585		—527

II	W_x	$\Delta R'$	$\Delta R'/R'$	$\Delta R/R'$	From	ΔR
	+50	+220	.0244	.0256	Table I	+230
	—50	—292	—.0324	—.0305	Table II	—275

III	ΔV	$\Delta R'$	$2\dfrac{\Delta R'}{R'}$	$-2\dfrac{\Delta V}{V}$	$\dfrac{\Delta R}{\Delta R'}-1$	$\dfrac{\Delta R}{\Delta R'}$	ΔR
	+150	+ 877	.1949	—.1333	.0616	1.062	+931
	+ 50	+ 305	.0678	—.0444	.0234	1.023	+312
	— 50	— 318	—.0707	+.0444	—.0263	.974	—310
	—150	—1002	—.2227	+.1333	—.0894	.911	—913

With these values of ΔR we may use the corrected range 9000 yards; and the corrections to be plotted opposite that corrected range may be interpolated from these values to intervals of 2% in C, 10 miles per hour in W_x and 10 f.s. in V.

TABLE I

Positive Changes in C and W_x.

$\Delta R'/R'$		$\Delta R/R'$ 0	1	2	3	4	5	6	7	8	9
.00	.0	000	010	020	031	041	051	061	071	082	092
.01	.0	102	113	123	134	144	155	165	176	186	197
.02	.0	208	219	230	241	252	263	274	285	296	308
.03	.0	319	331	342	354	365	377	388	400	411	423
.04	.0	435	447	459	471	483	495	507	519	531	543
.05	.0	556	568	581	593	606	619	631	644	656	669
.06	.0	683	696	709	722	735	749	762	775	788	801
.07	.0	815	828	842	856	870	884	898	912	926	940
.08	.0	954	968	982	997	*011	*026	*040	*054	*069	*083
.09	.1	098	113	128	143	158	174	189	204	219	234
.10	.1	250	265	281	297	313	329	345	361	377	393
.11	.1	409	425	442	458	474	491	507	524	540	557
.12	.1	574	591	608	626	643	661	678	696	713	731
.13	.1	748	766	784	802	820	838	856	874	892	910
.14	.1	929	948	967	986	*005	*024	*043	*062	*081	*100
.15	.2	119	138	157	177	197	217	237	257	277	297

Table II

Negative Changes in C and W_x.

						$\Delta R/R'$					
$\Delta R'/R'$		0	1	2	3	4	5	6	7	8	9
—.00	—.0	000	010	020	030	040	050	060	069	079	088
—.01	—.0	098	108	117	127	136	145	155	164	174	183
—.02	—.0	192	202	211	220	229	238	247	256	265	274
—.03	—.0	283	292	301	310	319	328	337	345	354	362
—.04	—.0	371	379	388	396	405	413	422	430	439	447
—.05	—.0	456	464	473	481	489	498	506	514	523	531
—.06	—.0	539	547	555	563	571	579	587	595	603	611
—.07	—.0	619	627	635	643	651	659	667	675	683	691
—.08	—.0	698	706	713	721	729	736	744	751	759	767
—.09	—.0	774	782	790	797	805	812	820	827	835	842
—.10	—.0	850	857	865	872	879	887	894	902	909	918
—.11	—.0	925	933	940	947	955	962	969	977	984	991
—.12	—.0	998	*006	*013	*020	*028	*036	*043	*050	*058	*065
—.13	—.1	072	080	087	094	102	109	117	124	131	138
—.14	—.1	145	153	160	167	175	182	190	197	204	212
—.15	—.1	219	227	234	242	249	257	264	271	279	286

APPENDIX IV.

THE WIND DEFLECTION FUNCTION.

Table I. is for the purpose of permitting the calculation of the wind deflection due to a cross-wind of W_y miles per hour at the same time as the range table itself after finding Z.

In Table I. a function designated D_w is given, and its value is such that the deflection in degrees is given by

$$\frac{ZWD_w{}^{\circ}}{V}$$

and for a ten mile wind we should have

$$\text{Deflection} = \frac{10ZD_w{}^{\circ}}{V}$$

Since the formula for wind deflection is

$$\text{Deflection} = \frac{84W}{V\cos\varphi}\left\{\frac{VT\cos\varphi}{X} - 1\right\}$$

we see that if $\cos\varphi$ in the denominator be taken as unity, we have,

$$\frac{ZWD_w}{V} = \frac{84W}{V}\left\{\frac{VT'}{Z} - 1\right\}$$

or

$$D_w = \frac{84}{Z}\left\{\frac{VT'}{Z} - 1\right\}$$

and from this and Ingalls' Table II. which give T' for tabular values of V and Z, Table I. was computed.

Strictly speaking the deflection as thus computed should be multiplied by $\sec\varphi$; but in direct fire this is scarcely necessary.

As an example of the application of Table I. we may compute the deflection for a 10 mile cross-wind for the 12-inch gun at 9000 yards.

Here $Z = 3366$, $V = 2250$.

Hence, from the table, for this V and Z

$$D_w = .00625$$

and

$$\text{Deflection} = \frac{3366}{2250} \times 10 \times .00625$$

$$= 0^{\circ}.093$$

As φ for this range is $6^{\circ}\ 45'$, $\sec\varphi = 1.007$ which would make no significant difference.

TABLE I.

Values of D_w in degrees, for use with the formula

$$\text{Deflection for W mile wind} = \frac{WZ}{V} \cdot D_w^{\circ}.$$

V	1600	1700	1800	1900	2000	2100
Z	°	°	°	°	°	°
0000	.0055	.0055	.0055	.0054	.0054	.0053
1000	.0059	.0058	.0058	.0057	.0057	.0056
2000	.0061	.0060	.0061	.0060	.0060	.0059
3000	.0061	.0062	.0063	.0063	.0063	.0062
4000	.0060	.0062	.0063	.0065	.0065	.0065
5000	.0059	.0061	.0064	.0066	.0067	.0067
6000	.0057	.0061	.0065	.0066	.0068	.0069
7000	.0055	.0060	.0064	.0065	.0068	.0070
8000	.0054	.0059	.0062	.0064	.0067	.0070
9000	.0053	.0057	.0060	.0063	.0066	.0069
10000	.0052	.0056	.0059	.0062	.0066	.0069
11000	.0051	.0055	.0058	.0061	.0065	.0068
12000	.0051	.0054	.0058	.0061	.0064	.0067
13000	.0050	.0054	.0057	.0061	.0063	.0067
14000	.0050	.0053	.0057	.0060	.0062	.0066
15000	.0049	.0053	.0056	.0059	.0062	.0065
16000	.0049	.0053	.0056	.0059	.0062	.0065
17000	.0049	.0052	.0055	.0058	.0061	.0064
18000	.0049	.0052	.0055	.0058	.0061	.0064
19000	.0048	.0052	.0055	.0057	.0060	.0063
20000	.0048	.0051	.0054	.0057	.0060	.0063
21000	.0048	.0051	.0054	.0057	.0060	.0062

TABLE I. (Continued)

Valves of D_w in degrees for use with the formula

$$\text{Deflection for W mile wind} = \frac{W\,Z}{V} \;.\; D_w{}^{\circ}.$$

v	2100	2200	2300	2400	2500	2600
z	°	°	°	°	°	°
0000	.0053	.0052	.0051	.0050	.0049	.0048
1000	.0056	.0056	.0055	.0054	.0053	.0052
2000	.0059	.0059	.0058	.0057	.0056	.0055
3000	.0062	.0062	.0061	.0060	.0059	.0059
4000	.0065	.0065	.0064	.0063	.0063	.0063
5000	.0067	.0068	.0067	.0067	.0067	.0067
6000	.0069	.0070	.0070	.0070	.0070	.0070
7000	.0070	.0071	.0072	.0072	.0072	.0072
8000	.0070	.0071	.0072	.0073	.0074	.0074
9000	.0069	.0071	.0072	.0074	.0075	.0076
10000	.0069	.0070	.0072	.0074	.0075	.0076
11000	.0068	.0070	.0071	.0073	.0075	.0076
12000	.0067	.0069	.0071	.0073	.0075	.0076
13000	.0067	.0069	.0071	.0073	.0074	.0076
14000	.0066	.0068	.0070	.0072	.0074	.0076
15000	.0065	.0068	.0070	.0072	.0074	.0076
16000	.0065	.0067	.0069	.0072	.0073	.0075
17000	.0064	.0067	.0069	.0071	.0073	.0075
18000	.0064	.0067	.0069	.0071	.0073	.0075
19000	.0063	.0066	.0069	.0071	.0073	.0075
20000	.0063	.0066	.0068	.0071	.0073	.0075
21000	.0062	.0065	.0068	.0071	.0073	.0075

TABLE I. (Continued)

Values of D_w in degrees for use with the formula

$$\text{Deflection for W mile wind} = \frac{WZ}{V} \cdot D_w^\circ$$

V	2600	2700	2800	2900	3000	3100
Z	°	°	°	°	°	°
0000	.0048	.0048	.0047	.0047	.0046	.0046
1000	.0052	.0051	.0050	.0050	.0049	.0048
2000	.0055	.0054	.0054	.0053	.0052	.0051
3000	.0059	.0058	.0057	.0056	.0055	.0054
4000	.0063	.0062	.0060	.0060	.0059	.0058
5000	.0067	.0066	.0064	.0063	.0063	.0062
6000	.0070	.0069	.0068	.0067	.0066	.0065
7000	.0072	.0072	.0072	.0071	.0070	.0069
8000	.0074	.0075	.0075	.0074	.0074	.0073
9000	.0076	.0077	.0077	.0077	.0077	.0077
10000	.0076	.0077	.0078	.0078	.0079	.0079
11000	.0076	.0078	.0079	.0079	.0080	.0081
12000	.0076	.0078	.0079	.0080	.0081	.0082
13000	.0076	.0078	.0079	.0080	.0081	.0082
14000	.0076	.0078	.0079	.0081	.0082	.0083
15000	.0076	.0078	.0079	.0081	.0082	.0083
16000	.0075	.0077	.0079	.0081	.0082	.0083
17000	.0075	.0077	.0079	.0081	.0082	.0084
18000	.0075	.0077	.0079	.0081	.0082	.0084
19000	.0075	.0077	.0079	.0081	.0083	.0084
20000	.0075	.0077	.0079	.0081	.0083	.0084
21000	.0075	.0077	.0079	.0081	.0083	.0084

TABLE I. (Continued)

Values of D_w in degrees for use with the formula

$$\text{Deflection due to W-mile wind} = \frac{W\,Z}{V} \cdot D_w{}^\circ$$

V	3100	3200	3300	3400	3500	3600
Z	°	°	°	°	°	°
0000	.0046	.0045	.0045	.0044	.0044	.0043
1000	.0048	.0047	.0047	.0046	.0046	.0045
2000	.0051	.0050	.0050	.0049	.0048	.0047
3000	.0054	.0053	.0053	.0052	.0051	.0050
4000	.0058	.0057	.0057	.0056	.0055	.0053
5000	.0062	.0061	.0060	.0059	.0058	.0056
6000	.0065	.0064	.0063	.0062	.0061	.0060
7000	.0069	.0068	.0067	.0066	.0065	.0064
8000	.0073	.0072	.0071	.0070	.0069	.0068
9000	.0077	.0076	.0075	.0075	.0073	.0073
10000	.0079	.0079	.0078	.0078	.0077	.0077
11000	.0081	.0081	.0081	.0081	.0080	.0080
12000	.0082	.0082	.0083	.0083	.0083	.0083
13000	.0082	.0083	.0084	.0084	.0085	.0085
14000	.0083	.0084	.0085	.0085	.0086	.0086
15000	.0083	.0084	.0085	.0086	.0087	.0088
16000	.0084	.0085	.0086	.0086	.0087	.0088
17000	.0084	.0085	.0086	.0087	.0088	.0089
18000	.0084	.0085	.0087	.0088	.0089	.0090
19000	.0084	.0086	.0087	.0088	.0089	.0090
20000	.0084	.0086	.0087	.0089	.0090	.0091
21000	.0084	.0086	.0087	.0089	.0090	.0091

APPENDIX V.

RELATED CHARGES FOR MORTARS

Considerations of economy, storage and freedom from confusion in loading must govern to a great extent in fixing mortar zones. The problem is best approached by ignoring these considerations in determining the first tentative zones, which should be such that the overlaps are ample; this is illustrated in Figure 2. The weight of powder charge, exclusive of igniter, should then be laid off horizontally, say 30 ounces to the inch and the velocity and pressure incident to its use should have vertical scales on the same sheet; the velocity, on the left, 50 f.s. to the inch; the pressure, on the right, 2000 pounds per square inch to the inch. Plot the curves of relation of charge-velocity and charge-pressure, from the firing data. Mark the velocity that corresponds to the limit of safe pressure (which find from the Ordnance Supply Manual or its amendments and Reports of Chief of Ordnance; *e.g.* 1904, 1905).

The problem may be generally stated thus:

To fulfil the following conflicting requirements:

1. To have the fewest number possible of different sized packages of powder.
2. To have certain packages, primed at both ends, for use as base charges.
3. To preserve adequate overlaps and to cover the field of fire effectively.
4. To avoid having too many small packages in a charge.
5. To prevent confusion in loading.

As a first step in the solution we select the most important zones covering the principal fighting ranges. Supposing these ranges for the 12-inch mortar to be 4000 to 9000 yards we have from Fig. 2, V = 810, 915, 1050.

An examination of the firing records shows that a slower powder was used for the outer zone with the 1046 lb. projectile and also for the 824 lb. projectile. It will, then, be necessary to find from the curve of the quick powder what velocity will be obtainable with the quick powder without exceeding the allowable pressure. This allowable pressure is taken to be 32000 lbs. per square inch.

The data give the following:

1046 lb. projectile.

$\tilde{\omega}$ = Charge	V	P = Pressure	Charge (ozs.)
$13\frac{9}{16}$ lbs.	560	8400	217
$16\frac{9}{16}$	625	9700	262
$25\frac{2}{16}$	800	17100	402
$33\frac{7}{16}$	950	27500	535

On prolonging the pressure and velocity curves it will be found that the maximum allowable pressure is reached for V = 1010 f.s. approximately. This gives an upper limit.

On taking the zones determined beforehand with 400 yard overlaps we find them to be

V	Zone limits	Charge
550	2970 to 2210	13.3 lbs.
600	3431 to 2600	15.25
660	4030 to 3070	17.9
725	4800 to 3631	21.25
810	5940 to 4429	25.7
915	7476 to 5520	31.375
1050	9250 to 7027	

If now we note that the following charges and muzzle velocities do not differ materially from the above and are susceptible of a simple subdivision, it is seen that the problem is solved:

V	Charge
556	13.5 = A
608	15.625 = B
666	18.25 = A + C *or* B + ½ C
718	20.875 = B + C
817	26.125 = B + 2 C
915	31.375 = B + 3 C
1005	36.625 = B + 4 C

in which

A = 13.5 primed at both ends
B = 15.625 primed at both ends
C = 5.25 not primed
½C = 2.625 not primed

Thus the powder for the zones employing the 1046 pound projectile may be put up in 4 packages. The 824 pound projectile needs for the higher velocity, (which will have to be reduced to 1240 f.s. to give the overlap necessary), a different powder to avoid excessive pressures.

The employment of Fig. 2 will show the new zone limits to be as follows :

V	Limits		Overlaps	Charge
556	3055	2257	395	A
608	3511	2660	392	B or A + ½ C
666	4094	3119	524	B+ ½ C *or* A + C
718	4717	3570	195	B + C
817	6065	4522	545	B + 2 C
915	7476	5520	951	B + 3 C
1005	8633	6525	425	B + 4 C
1240	11265	8208		

If a different powder were used the above arrangement would not hold and hence this particular powder or its equivalent must be used.

This solution is not by any means the only one, as is evident. The package A might be eliminated for instance by doing away with the 556 f.s. zone and compensating therefor by running the elevations in the 608 f.s. zone up to 70°, in case of emergency. The drift would be erratic above 65°, but this zone is not of great importance, comparatively. This would reduce the inner range to about 2250 yards as before with one less zone, one less package and one less base charge.

If the outer range must be as high as 12000 yards two zones could be used with the slower power not herein dealt with numerically but only referred to. We would thus have a base charge and a secondary charge for this slower powder.

Taking as a starting point that velocity for the quicker powder which would give the 1046-pound projectile a sufficient overlap with the inner of the two zones, a proceeding similar to that used in the numerical illustration above should be followed decreasing the charge by equal parts (C, above, or half C), to cover the field most economically.

The intention here is but to indicate the method to be followed; and the character of powders finally used will depend upon what may be found to give the best arrangements of charges, etc., based upon the best of the solutions possible with available powders.

APPENDIX VI.

THE REDUCING FACTOR AND THE NECESSITY FOR ITS USE

Calculations of range-tables on the basis that

$$C = f_a \cdot \frac{w}{cd^2}$$

do not give results in accord with practice.

Experimental firings at Sandy Hook with the 3-inch gun 1700 f.s., 6-inch gun 3000 f.s., 12-inch gun 2250 f.s. and 2500 f.s. and the 12-inch mortars all indicate that the total curvature of the trajectory seriously affects the flight of the projectile, and as between elevations for different ranges with the same gun, projectile and muzzle velocity, a coefficient of curvature introduced into the denominator of the ballistic coefficient the value of which coefficient is $1 + \frac{4}{3}\left(\frac{\varphi^\circ + \omega^\circ}{180^\circ}\right)$ or $1 + \left(\frac{\varphi + \omega}{135}\right)$ will make a constant coefficient of form answer all purposes.

The coefficient of form c refers to the head and its junction with the body of the projectile.

As between projectiles of different cubic density, $\frac{w}{d^3}$, it is noted that in direct fire a further coefficient which is twice the reciprocal of the cubic density is needed in the denominator in order to pass from one kind of projectile to the other.

The reducing factor then becomes

$$\frac{d^3}{2w}\left(1 + \frac{\varphi + \omega}{135}\right) = \gamma = \frac{d^3}{2w} \cdot \gamma'$$

We may say, then, that the coefficient of curvature, γ', is unity at zero range and increases by a fraction which is $\frac{1}{135}$ of the total curvature of the trajectory, expressed in degrees or $\frac{4}{3\pi}$ times the curvature expressed in radians.

The great difficulty up to the present time has been to obtain a constant projectile coefficient; and the isolation of the curvature coefficient γ' has made this possible, so that a group of shots at extreme range with a certain gun and projectile should now determine the matter.

It must be borne in mind that projectiles have peculiarities which are not determinable beforehand. Thus a shrapnel, a C.I. shot and an A.P. shell will all have slightly different values of k on account of internal characteristics. This k, as already indicated in Part I, must be found from experiment. A single group of shots at long range should determine it, or, what amounts to the same thing, determine the coefficient of form c.

The value of c for the 3-inch field gun, V = 1700, the 6-inch gun, V = 3000, the 12-inch V = 2250 and V = 2500 with service projectiles is practically unity at all elevations. Prior to the use of the reducing factor the c then used was different for each elevation and for projectiles of different cubic density.

APPENDIX VII.

CURVATURE OF THE EARTH

If a point move away on the surface of a sphere from another point, it will, after moving 90° on a great circle, have acquired an angular depression of 45° from the tangent to the circle at the stationary point. When it has reached a point 180° from the stationary point it will be diametrically opposite the stationary point and a line joining the two points will be at right angles to the tangent to the circle at the stationary point ; and, in general, a simple figure will show that the depression due to curvature is half the curvature of the circle between the two points. There is thus a depression angle of ½ of a minute for a curvature of the earth of 1′. Now 1′ is equal to a knot or 1.152 miles. That is to say 2027 yards. We thus have a depression due to curvature of 1′ for 4054 yards, or, 0′.2467 per 1000 yards. That is 0′.987 or, practically, 1′ per 4000 yards. The curvature in feet may be found if desired from this relation.

It becomes

$$K = [3{\cdot}3333 - 10]\ R^2 = .2154\left(\frac{R}{1000}\right)^2.$$

in which K is the curvature in feet and R the range in yards.

The curvature in feet for ranges at 1000 yards interval are given in the following table.

CURVATURE IN FEET

Range	Curvature	△	Range	Curvature	△
yds.	feet	feet	yds.	feet	feet
1000	0.22	0.64	12000	31.02	5.38
2000	0.86	1.08	13000	36.40	5.82
3000	1.94	1.51	14000	42.22	6.25
4000	3.45	1.94	15000	48.47	6.67
5000	5.39	2.36	16000	55.14	7.11
6000	7.75	2.80	17000	62.25	7.54
7000	10.55	3.24	18000	69.79	7.97
8000	13.79	3.66	19000	77.76	8.40
9000	17.45	4.09	20000	86.16	8.83
10000	21.54	4.52	21000	94.99	9.26
11000	26.06	4.96	22000	104.25	

APPENDIX VIII.

I. RANGE CORRECTIONS FOR HIGH ANGLE FIRE

The quadratic law of resistance for low velocities affords the hypothesis that the retardation is small in comparison with the acceleration due to gravity. This is especially true with the very heavy projectiles employed in seacoast mortars such as our 12-inch mortars.

These suppositions give, by limiting an expansion by Stirling's formula to its significant terms, an equation of the form

$$\frac{V^2 \sin 2\varphi}{gX} = 1 + \frac{A'}{C} V^2 \qquad (1)$$

in which, for the quadratic law, $Cr = AV^2$, A' is a function of only one variable, and that is φ.

Furthermore, actual firings with mortars, of which data of a large number of shots are at hand, show that A' varies very slowly with φ. In any case where φ is assumed and V or C varies, this latter does not signify, as A' will remain fixed so long as φ does not change. But for variations in φ it may be assumed without significant error that A' is constant where we are seeking the resulting change in the range.

Equation (1) thus affords the basis of discussion of our range variations due to variations in V, C or φ, respectively.

Allowing V, φ, X and C to vary we have

$$\frac{V^2 \sin 2\varphi}{gX}\left\{\frac{d \sin 2\varphi}{\sin 2\varphi} + \frac{2dV}{V} - \frac{dX}{X}\right\} = \frac{V^2A'}{C}\left\{\frac{dA'}{A'} - \frac{dC}{C} + \frac{2dV}{V}\right\}.$$

and since, from Eq. (1), $\frac{V^2A'}{C} = \frac{V^2 \sin 2\varphi}{gX} - 1$,

we have, on dividing through by $\frac{V^2 \sin 2\varphi}{gX}$,

$$\frac{d \sin 2\varphi}{\sin 2\varphi} + \frac{2dV}{V} - \frac{dX}{X} = \left(1 - \frac{gX}{V^2 \sin 2\varphi}\right)\left(\frac{dA'}{A'} - \frac{dC}{C} + \frac{2dV}{V}\right).$$

From this equation we find.

$$\frac{dX}{X} = \frac{d \sin 2\varphi}{\sin 2\varphi} + \frac{2gX}{V^2 \sin 2\varphi} \cdot \frac{dV}{V} + \left(1 - \frac{gX}{V^2 \sin 2\varphi}\right)\left(\frac{dC}{C} - \frac{dA'}{A'}\right)$$

Whence the formulas of Chapter VII, Part I, are derived. Thus, replacing d by $\triangle$ for small increments,

(1) X and C varying,

$$\frac{\triangle X}{X}=\frac{\triangle R}{R}=\left(1-\frac{gX}{V^2\sin 2\varphi}\right)\frac{\triangle C}{C} \qquad \text{(I)}$$

(2) X and V varying,

$$\frac{\triangle X}{X}=\frac{\triangle R}{R}=\frac{2gX}{V^2\sin 2\varphi}\cdot\frac{\triangle V}{V} \qquad \text{(II)}$$

If A′ be taken as constant between 45° and 65° we have $\frac{dA'}{A'}=0$, and hence

$$\frac{dX}{X}=\frac{d\sin 2\varphi}{\sin 2\varphi}+\frac{2gX}{V^2\sin 2\varphi}\cdot\frac{dV}{V}+\left(1-\frac{gX}{V^2\sin 2\varphi}\right)\frac{dC}{C}.$$

From which

$$\frac{\triangle X}{X}=\frac{\triangle R}{R}=\frac{\triangle\sin 2\varphi}{\sin 2\varphi}$$

which is best put into the form

$$\triangle\log R=\triangle\log\sin 2\varphi \qquad \text{(III)}$$

To illustrate the use of (III),

$V=600$; $\varphi=52°$; $R=3333.3$ to find R for $\varphi=55°$.

$$\log\sin 110° = 9.97298-10 = \log\sin 70°$$
$$\text{a.c. }\log\sin 104° = 0.01310-10 = \text{a.c. }\log\sin 76°$$

$$\triangle\log\sin 2\varphi = 9.98608-10 = \triangle\log R$$
$$3.52288 = \log R$$
$$3.50896 = \log R'$$
$$3228 = \log R'$$

Hence R′ = 3228 yards, and $\triangle R = -105$ yards.

2. HIGH ANGLE FIRE RATIOS

This table is based on the quadratic law of resistance

$$r_1 = Cr = Av^2 = .00004676\ v^2$$

Various forms of tables have been computed based on Euler's solution of the problem, among which may be mentioned those of Graevenitz and Otto. These tables were later extended by Lardillon to include lower angles of departure; and by Siacci to include higher velocities. Ingalls in Table IV, Artillery Circular M, introduces the ballistic coefficient into the ratios.

In the table given herein the ratios $\frac{10X}{V^2}$, $\frac{10T}{V}$, ω, $\frac{v_\omega}{V}$ and $\frac{y_0}{X}$ are tabulated with $\frac{V}{\sqrt{C}}$ as argument and on pages at the top of each of which is given the value of φ pertaining to that page.

The quantity log h/X is given in order that h, the mean height of the trajectory may be found in order to obtain the altitude factor.

Ordinarily where actual firings are made to obtain data for a range-table, the values of V, φ, X and T are recorded. It then remains only to correct for abnormal conditions and with the resulting values of $\frac{10X}{V^2}$ find ω and v_ω, from which the drift and perforation can be found, the drift constant being determined by the observed drifts.

In the absence of tables the following relations, which are true with close approximation, may be used for this purpose:

$$\tan \omega = \frac{gT^2}{X} - \tan \varphi$$

$$\frac{v_\omega \ T \cos \omega}{X} = \frac{X}{V \ T \cos \varphi} .$$

$$y_0 = \frac{gT^2}{8}$$

If there are no preliminary firings and a range table is to be computed, then it is best to take sec φ as a reducing factor in computing C and $\frac{V}{\sqrt{C}}$. Thus, for a range-table,

$$C = f_a \ . \ \frac{w}{c \, d^2 \sec \varphi}$$

in which

$$f_a = \frac{1}{1 - .000027 \, h}$$

h being found from log h/X which, in turn, is found from the table of ratios by the use of a C in which f_a is taken as unity as a first approximation.

This process may be repeated if necessary. $\frac{10X}{V^2}$ is found at the same time as log h/X, from the first approximation to $\frac{V}{\sqrt{C}}$ and thus h may be calculated since h/X and X are known.

Note on time of flight.

If the time of flight is not observed it may be found by the formula :

$$T = \frac{2V \sin \varphi}{g_1}$$

in which

$$g_1 = 32.16 + 8 \left\{ 1 - \frac{gX}{V^2 \sin 2\varphi} \right\}.$$

This gives results in close accord with actual firings.

3. ELEVATION FOR A MAXIMUM RANGE

In plotting the range-elevation curves of Fig. 1, Chapter VII, it is desirable to know everything possible about the general form of the curve. As shown (page 40) the curve is, for a given V, nearly of the general form $X = k' \sin 2\varphi$, in which k' is taken as an arbitrary constant the mean value of which is separately determined from the firings *in each zone.*

Now k' is *not* strictly constant in each zone but varies slightly with the elevation in such a manner that the greatest range occurs below 45°, and that the mean value of k' gives ranges somewhat incorrectly at 45° and 65°.

For the quadratic law of resistance the elevation for the greatest range fulfils a law of the form

$$\sec^2 \varphi = 2 - k'' \frac{V^2}{C} \qquad (1)$$

Using $\sec \varphi$ as a reducing factor and determining k'' from the firings for the 12-inch mortar, 1046-pound projectile, we have

$$R(\varphi) = 2 \cos \varphi - \sec \varphi = \frac{d^2}{w} \cdot \left(\frac{V}{1000} \right)^2 \qquad (2)$$

The following table gives the values of $R(\varphi)$.

φ °	$R(\varphi)$	φ °	$R(\varphi)$	φ °	$R(\varphi)$	φ °	$R(\varphi)$
30	.58	34	.45	38	.31	42	.14
31	.55	35	.42	39	.27	43	.10
32	.52	36	.38	40	.23	44	.05
33	.49	37	.34	41	.18	45	.00

Application, to the 12″ mortar, 1046-lb. projectile, of formula (2)

V	$R(\varphi)$	φ
500	.034	44°.3
600	.050	44°.0
700	.067	43°.7
800	.088	43°.3
900	.112	42°.7
1000	.138	42°.0
1100	.167	41°.3

In differencing as on page 38 the first differences should vanish for the values of φ given, to within a fraction of a degree.

$\varphi = 45°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{x}$	$\frac{y_0}{X}$
			°	′		—10	
100	.439	.308	45	27	.983	9.203	.252
110	.439	.307	45	29	.982	9.203	.252
120	.439	.306	45	32	.980	9.203	.252
130	.438	.305	45	35	.978	9.204	.252
140	.438	304	45	38	.976	9.204	.253
150	.437	.303	45	42	.974	9.205	.253
160	.437	.302	45	47	.971	9.205	.253
170	.436	.301	45	52	.968	9.206	.254
180	.435	.300	45	58	.965	9.206	.254
190	.435	.299	46	04	.961	9.207	.255
200	.434	.297	46	11	.957	9.208	.255
210	.434	.296	46	18	.953	9.208	.256
220	.433	.295	46	25	.949	9.209	.256
230	.433	.294	46	32	.945	9.210	.257
240	.432	.292	46	40	.941	9.211	.257
250	.431	.290	46	48	.937	9.212	.258
260	.431	.289	46	55	.932	9.213	.259
270	.430	.288	47	03	.927	9.214	.259
280	.429	.286	47	11	.922	9.215	.260
290	.429	.284	47	19	.917	9.216	.261
300	.428	.282	47	28	.912	9.217	.261
310	.427	.281	47	37	.907	9.218	.262
320	.427	.279	47	46	.902	9.219	.263
330	.426	.277	47	56	.897	9.220	.264
340	.425	.275	48	06	.892	9.221	.264
350	.425	.274	48	16	.887	9.222	.265
360	.424	.272	48	26	.881	9.224	.266
370	.423	.270	48	36	.876	9.225	.267
380	.422	.268	48	47	.871	9.226	.268
390	.421	.266	48	57	.866	9.227	.269
400	.420	.264	49	08	.860	9.229	.269
410	.419	.263	49	19	.855	9.230	.270
420	.418	.261	49	30	.850	9.231	.271
430	.417	.259	49	41	.844	9.232	.272
440	.416	.257	49	52	.838	9.234	.273
450	.415	.255	50	03	.833	9.235	.274

$\varphi = 45^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{x}$	$\frac{y_0}{X}$
			°	′		—10	
460	.414	.253	50	14	.827	9.237	.275
470	.413	.251	50	25	.821	9.238	.275
480	.412	.249	50	37	.815	9.240	.276
490	.411	.247	50	48	.809	9.241	.277
500	.410	.245	51	00	.803	9.242	.278
510	.409	.243	51	12	.798	9.244	.279
520	.408	.241	51	24	.792	9.245	.280
530	.407	.239	51	35	.787	9.247	.280
540	.406	.237	51	47	.781	9.248	.281
550	.405	.235	51	58	.776	9.250	.282
560	.404	.233	52	10	.770	9.251	.283
570	.403	.231	52	22	.764	9.253	.284
580	.402	.229	52	34	.759	9.254	.285
590	.401	.227	52	45	.753	9.255	.286
600	.400	.225	52	57	.748	9.256	.287
610	.398	.223	53	09	.743	9.258	.288
620	.397	.221	53	21	.738	9.259	.289
630	.396	.220	53	33	.733	9.260	.290
640	.395	.218	53	44	.728	9.262	.291
650	.394	.216	53	56	.723	9.263	.292
660	.393	.214	54	07	.718	9.264	.293
670	.392	.212	54	19	.713	9.266	.294
680	.391	.210	54	30	.708	9.267	.295
690	.390	.208	54	42	.703	9.268	.296
700	.389	.206	54	53	.698	9.269	.297
710	.387	.205	55	05	.693	9.271	.298
720	.386	.203	55	16	.688	9.272	.299
730	.385	.201	55	27	.684	9.273	.300
740	.384	.199	55	38	.679	9.275	.301
750	.383	.197	55	50	.674	9.276	.302
760	.382	.196	56	01	.670	9.277	.303
770	.381	.194	56	12	.665	9.279	.304
780	.380	.192	56	23	.660	9.280	.305
790	.379	.190	56	34	.656	9.281	.306
800	.378	.188	56	45	.651	9.282	.307

$\varphi = 46°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.447	.307	46	28	.983	9.216	.261
110	.447	.306	46	30	.982	9.216	.261
120	.446	.305	46	33	.980	9.217	.261
130	.446	.304	46	36	.978	9.217	.261
140	.445	.303	46	39	.976	9.218	.262
150	.445	.302	46	43	.974	9.218	.262
160	.444	.301	46	48	.971	9.219	.262
170	.444	.300	46	53	.968	9.219	.263
180	.443	.299	46	59	.965	9.220	.263
190	.443	.298	47	05	.961	9.220	.264
200	.442	.297	47	11	.957	9.221	.264
210	.442	.295	47	18	.953	9.221	.265
220	.441	.294	47	25	.949	9.222	.265
230	.441	.293	47	32	.945	9.223	.266
240	.440	.291	47	40	.941	9.224	.266
250	.439	.290	47	48	.937	9.225	.267
260	.439	.289	47	55	.932	9.226	.268
270	.438	.287	48	03	.927	9.227	.268
280	.437	.286	48	11	.922	9.228	.269
290	.436	.284	48	19	.917	9.229	.270
300	.435	.282	48	28	.912	9.230	.270
310	.435	.281	48	37	.907	9.231	.271
320	.434	.279	48	46	.902	9.233	.272
330	.433	.277	48	56	.897	9.234	.273
340	.432	.275	49	06	.892	9.235	.273
350	.431	.273	49	16	.887	9.236	.274
360	.430	.271	49	26	.882	9.237	.275
370	.429	.269	49	36	.876	9.239	.276
380	.429	.267	49	47	.871	9.240	.277
390	.428	.265	49	57	.866	9.241	.278
400	.427	.263	50	08	.861	9.242	.278
410	.426	.262	50	19	.856	9.244	.279
420	.425	.260	50	30	.850	9.245	.280
430	.424	.258	50	41	.845	9.246	.281
440	.423	.256	50	52	.839	9.247	.282
450	.422	.254	51	03	.834	9.249	.283

$\varphi = 46°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.421	.252	51	14	.828	9.250	.284
470	.420	.250	51	25	.822	9.251	.285
480	.419	.248	51	37	.816	9.253	.286
490	.418	.246	51	48	.810	9.254	.287
500	.417	.244	52	00	.804	9.255	.288
510	.416	.242	52	11	.798	9.257	.289
520	.415	.240	52	23	.792	9.258	.290
530	.414	.238	52	35	.787	9.259	.290
540	.413	.236	52	47	.781	9.261	.291
550	.412	.234	52	58	.776	9.262	.292
560	.411	.232	53	09	.771	9.263	.293
570	.410	.230	53	21	.765	9.265	.294
580	.409	.228	53	33	.760	9.266	.295
590	.408	.226	53	44	.755	9.268	.296
600	.407	.224	53	56	.750	9.269	.297
610	.405	.223	54	08	.745	9.270	.298
620	.404	.221	54	20	.740	9.272	.300
630	.403	.219	54	32	.735	9.273	.301
640	.402	.217	54	43	.730	9.274	.302
650	.401	.215	54	54	.725	9.276	.303
660	.400	.213	55	05	.720	9.277	.304
670	.399	.211	55	17	.715	9.278	.305
680	.398	.209	55	28	.710	9.280	.306
690	.397	.207	55	40	.705	9.281	.307
700	.395	.205	55	51	.700	9.282	.308
710	.394	.204	56	03	.695	9.284	.309
720	.393	.202	56	14	.690	9.285	.310
730	.392	.200	56	25	.686	9.286	.311
740	.391	.198	56	36	.681	9.288	.312
750	.390	.196	56	48	.676	9.289	.313
760	.389	.194	56	59	.672	9.290	.314
770	.387	.192	57	10	.667	9.292	.315
780	.386	.191	57	21	.662	9.293	.316
790	.385	.189	57	32	.658	9.294	.317
800	.384	.187	57	42	.653	9.296	.318

$\varphi = 47°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	'		—10	
100	.454	.306	47	29	.983	9.229	.270
110	.454	.305	47	31	.982	9.229	.270
120	.454	.304	47	34	.980	9.230	.270
130	.453	.303	47	37	.978	9.230	.270
140	.453	.302	47	40	.976	9.231	.271
150	.452	.301	47	44	.974	9.231	.271
160	.452	.300	47	48	.971	9.232	.271
170	.451	.299	47	53	.968	9.232	.272
180	.450	.298	47	58	.965	9.233	.272
190	.450	.297	48	04	.961	9.234	.273
200	.449	.296	48	11	.957	9.235	.273
210	.449	.294	48	18	.953	9.235	.274
220	.448	.293	48	25	.949	9.236	.274
230	.448	.292	48	32	.945	9.237	.275
240	.447	.290	48	40	.941	9.238	.276
250	.446	.289	48	48	.937	9.239	.276
260	.446	.287	48	56	.933	9.240	.277
270	.445	.286	49	04	.928	9.241	.278
280	.444	.284	49	12	.923	9.242	.278
290	.443	.282	49	20	.918	9.243	.279
300	.442	.281	49	29	.913	9.244	.280
310	.442	.279	49	38	.908	9.245	.281
320	.441	.277	49	47	.903	9.247	.281
330	.440	.275	49	56	.898	9.248	.282
340	.439	.273	50	06	.893	9.249	.283
350	.438	.272	50	16	.888	9.250	.284
360	.437	.270	50	25	.883	9.251	.284
370	.436	.268	50	35	.877	9.253	.285
380	.436	.266	50	46	.872	9.254	.286
390	.435	.264	50	57	.867	9.255	.287
400	.434	.262	51	08	.862	9.256	.288
410	.433	.261	51	18	.857	9.258	.289
420	.432	.259	51	28	.851	9.259	.290
430	.431	.257	51	38	.846	9.260	.291
440	.430	.255	51	49	.840	9.261	.292
450	.429	.253	52	00	.835	9.263	.293

$\varphi = 47°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.428	.251	52	11	.829	9.294	.294
470	.427	.249	52	23	.823	9.265	.295
480	.426	.247	52	35	.817	9.267	.296
490	.425	.245	52	47	.811	9.268	.297
500	.424	.243	52	59	.805	9 269	.298
510	.423	.241	53	11	.800	9.270	.299
520	.422	.239	53	23	.794	9.272	.300
530	.421	.237	53	34	.788	9.273	.301
540	.420	.235	53	46	.783	9.274	.302
550	.419	.233	53	57	.777	9.276	.303
560	.418	.231	54	08	.772	9.277	.304
570	.417	.229	54	19	.767	9.278	.305
580	.416	.227	54	31	.762	9.279	.306
590	.415	.225	54	42	.757	9.281	.307
600	.414	.223	54	54	.752	9.282	.308
610	.412	.222	55	06	.747	9.283	.309
620	.411	.220	55	18	.742	9.284	.310
630	.410	.218	55	30	.737	9.286	.312
640	.409	.216	55	41	732	9.287	.313
650	.407	.214	55	52	.727	9.288	.314
660	.406	.212	56	03	.722	9.289	.315
670	.405	.210	56	15	.717	9.291	.316
680	.403	.208	56	26	.712	9.292	.317
690	.402	.206	56	37	.707	9.293	.318
700	.401	.204	56	48	.702	9.295	.319
710	.400	.203	57	00	.698	9.296	.320
720	.398	.201	57	11	.693	9.297	.321
730	.397	.199	57	21	.688	9.299	.322
740	.396	.197	57	32	.683	9.300	.323
750	.395	.195	57	44	.679	9.301	.324
760	.394	.193	57	55	.674	9.303	.325
770	.393	.192	58	06	.669	9.304	.326
780	.392	.190	58	16	.665	9.306	.327
790	.391	.188	58	27	.660	9.308	.328
800	.390	.186	58	38	.655	9.309	.329

$\varphi = 48^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		−10	
100	.461	.305	48	29	.984	9.243	.280
110	.461	.304	48	31	.983	9.243	.280
120	.460	.303	48	34	.981	9.244	.280
130	.460	.302	48	37	.979	9.244	.280
140	.459	.301	48	41	.977	9.245	.281
150	.459	.300	48	45	.975	9.245	.281
160	.458	.299	48	49	.972	9.246	.281
170	.458	.298	48	53	.969	9.246	.282
180	.457	.297	48	58	.966	9.247	.282
190	.457	.296	49	04	.962	9.248	.283
200	.456	.295	49	01	.958	9.249	.283
210	.456	.293	49	18	.954	9.249	.284
220	.455	.292	49	25	.950	9.250	.284
230	.454	.291	49	32	.946	9.251	.285
240	.454	.289	49	40	.942	9.252	.286
250	.453	.288	49	48	.938	9.253	.286
260	.452	.286	49	56	.934	9.254	.287
270	.452	.285	50	04	.929	9.255	.288
280	.451	.283	50	12	.924	9.256	288
290	.450	.282	50	20	.919	9.257	.289
300	.449	.280	50	29	.914	9.258	.290
310	.449	.278	50	38	.909	9.259	.291
320	.448	.276	50	47	.904	9.261	.291
330	.447	.274	50	56	.899	9.262	.292
340	.446	.273	51	06	.894	9.263	.293
350	.445	.271	51	16	.889	9.264	.294
360	.445	.269	51	25	.884	9.265	.294
370	.444	.267	51	35	.878	9.267	.295
380	.443	.265	51	45	.873	9.268	296
390	.442	.263	51	56	.868	9.269	297
400	.441	.261	52	07	.863	9.270	.298
410	.440	.260	52	17	.858	9.272	.299
420	.439	.258	52	28	.852	9.273	.300
430	.438	.256	52	39	.847	9.274	.301
440	.437	.254	52	50	.842	9.275	.302
450	.436	.252	53	01	.836	9.277	.303

$\varphi = 48°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.435	.250	53	12	.830	9.278	.304
470	.434	.248	53	23	.824	9.279	.305
480	.433	.246	53	35	.819	9.281	.306
490	.432	.244	53	47	.813	9.282	.307
500	.431	.242	53	58	.807	9.283	.309
510	.430	.240	54	10	.802	9.284	.310
520	.429	.238	54	22	.796	9.286	.311
530	.428	.236	54	33	.790	9.287	.312
540	.427	.234	54	44	.785	9.288	.313
550	.426	.232	54	55	.779	9.289	.314
560	.424	.230	55	06	.774	9.291	.315
570	.423	.228	55	17	.769	9.292	.316
580	.422	.226	55	29	.764	9.293	.317
590	.521	.224	55	40	.759	9.294	.318
600	.420	.222	55	52	.754	9.296	.319
610	.418	.221	56	04	.749	9.297	.321
620	.417	.219	56	16	.744	9.298	.322
630	.416	.217	56	27	.739	9.299	.323
640	.415	.215	56	38	.734	9.301	.324
650	.413	.213	56	49	.729	9.302	.325
660	.412	.211	57	00	.724	9.303	.326
670	.411	.209	57	12	.719	9.305	.327
680	.410	.207	57	23	.714	9.306	.328
690	.408	.205	57	34	.709	9.307	.329
700	.407	.203	57	45	.704	9.309	.330
710	.406	.202	57	56	.700	9.310	.332
720	.405	.200	58	07	.695	9.311	.333
730	.404	.198	58	17	.690	9.313	.334
740	.403	.196	58	28	.685	9.314	.335
750	.401	.194	58	39	.681	9.315	.336
760	.400	.192	58	50	.676	9.317	.337
770	.399	.191	59	01	.671	9.318	.338
780	.398	.189	59	11	.667	9.320	.339
790	.397	.187	59	22	.662	9.322	.340
800	.396	.185	59	33	.657	9.323	.341

$\varphi = 49^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.468	.304	49	30	.984	9.257	.290
110	.468	.303	49	32	.982	9.257	.290
120	.468	.302	49	35	.981	9.257	.290
130	.467	.301	49	38	.979	9.258	.290
140	.467	.300	49	42	.977	9.258	.291
150	.466	.299	49	46	.975	9.259	.291
160	.466	.298	49	51	.972	9.259	.291
170	.465	.297	49	55	.970	9.260	.292
180	.465	.296	50	01	.966	9.261	.292
190	.464	.295	50	06	.962	9.262	.293
200	.463	.294	50	11	.958	9.263	.293
210	.463	.292	50	18	.955	9.263	.294
220	.462	.291	50	24	.951	9.264	.295
230	.461	.290	50	31	.947	9.265	.295
240	.461	.288	50	39	.943	9.266	.296
250	.460	.287	50	47	.939	9.257	.297
260	.459	.285	50	55	.935	9.268	.297
270	.458	.284	51	03	.930	9.269	.298
280	.457	.282	51	12	.925	9.270	.299
290	.457	.281	51	20	.920	9.271	.300
300	.456	.279	51	28	.915	9.272	.300
310	.455	.277	51	37	.910	9.273	.301
320	.454	.275	51	46	.905	9.275	.302
330	.453	.273	51	55	.900	9.276	.303
340	.452	.271	52	05	.895	9.277	.303
350	.451	.270	52	15	.890	9.278	.304
360	.451	.268	52	25	.885	9.279	.305
370	.450	.266	52	34	.880	9.281	.306
380	.449	.264	52	44	.875	9.282	.307
390	.448	.262	52	55	.870	9.283	.308
400	.448	.260	53	06	.864	9.284	.309
410	.447	.259	53	17	.859	9.286	.310
420	.446	.257	53	28	.854	9.287	.311
430	.445	.255	53	38	.849	9.288	.312
440	.444	.253	53	49	.844	9.290	.313
450	.443	.251	53	59	.838	9.291	.314

$\varphi = 49^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{V_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	'		—10	
460	.442	.249	54	10	.832	9.292	.315
470	.441	.247	54	21	.826	9.293	.316
480	.440	.245	54	33	.820	9.295	.317
490	.438	.243	54	44	.814	9.296	.318
500	.437	.241	54	56	.808	9.297	.320
510	.436	.239	55	07	.803	9.299	.321
520	.435	.237	55	18	.797	9.300	.322
530	.434	.235	55	29	.791	9.301	.323
540	.433	;233	55	40	.786	9.232	.324
550	.432	.231	55	52	.781	9.304	.325
560	.430	.229	56	03	.776	9.305	.326
570	.429	.227	56	14	.771	9.306	.327
580	.428	.225	56	26	.766	9.307	.328
590	.427	.223	56	37	.761	9.309	.329
600	.426	.221	56	48	.756	9.310	.330
610	.424	.220	57	00	.751	9.311	.332
620	.423	.218	57	12	.746	9.312	.333
630	.422	.216	57	23	.741	9.314	.344
640	.420	.214	57	34	.736	9.315	.335
650	.419	.212	57	45	.731	9.316	.336
660	.417	.210	57	56	.726	7.318	.337
670	.416	.208	58	07	.721	9.319	.338
680	.415	.206	58	18	.716	9.320	.339
690	.414	.204	58	29	.711	9.322	.340
700	.413	.202	58	40	.706	9.323	.342
710	.411	.201	58	51	.702	9.324	.343
720	.410	.199	59	01	.697	9.326	.344
730	.409	.197	59	12	.692	9.327	.345
740	.408	.195	59	23	.687	9.328	.346
750	.407	.193	59	34	.683	9.330	.347
760	.406	.191	59	44	.678	9.331	.348
770	.405	.190	59	44	.673	9.332	.350
780	.404	.188	60	05	.669	9.334	.351
790	.403	.186	60	16	.664	9.335	.352
800	.402	.184	60	26	.659	9.337	.353

$\varphi = 50^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.475	.302	50	30	.985	9.271	.300
110	.475	.301	50	33	.983	9.271	.300
120	.475	.300	50	36	.981	9.272	.300
130	.474	.299	50	39	.979	9.272	.301
140	.474	.298	50	43	.977	9.273	.301
150	.473	.297	50	47	.974	9.273	.302
160	.473	.296	50	52	.971	9.274	.302
170	.472	.295	50	56	.968	9.274	.303
180	.472	.294	51	01	.965	9.275	.303
190	.471	.293	51	06	.962	9.276	.304
200	.470	.292	51	11	.959	9.277	.304
210	.470	.290	51	17	.955	9.278	.305
220	.469	.289	51	23	.951	9.279	.305
230	.468	.288	51	30	.947	9.280	.306
240	.468	.286	51	38	.943	9.280	.307
250	.467	.285	51	46	.939	9.281	.307
260	.466	.283	51	54	.935	9.282	.308
270	.465	.282	52	02	.931	9.283	.309
280	.464	.280	52	11	.926	9.284	.309
290	.464	.278	52	19	.921	9.285	.310
300	.463	.277	52	28	.916	9.286	.311
310	.462	.275	52	37	.911	9.288	.311
320	.461	.274	52	46	.906	9.289	.312
330	.460	.272	52	55	.901	9.290	.313
340	.459	.270	53	05	.896	9.291	.314
350	.458	.268	53	14	.891	9.292	.315
360	.458	.266	53	24	.886	9.293	.316
370	.457	.264	53	33	.881	9.295	.317
380	.456	.262	53	43	.876	9.296	.318
390	.455	.260	53	53	.871	9.297	.319
400	.454	.258	54	04	.866	9.298	.320
410	.453	.257	54	15	.861	9.300	.321
420	.452	.255	54	26	.856	9.301	.322
430	.451	.253	54	36	.851	9.303	.323
440	.450	.251	54	47	.846	9.305	.324
450	.448	.249	54	57	.840	9.306	.325

$\varphi = 50°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.447	.247	55	08	.834	9.307	.327
490	.446	.245	55	19	.828	9.308	.328
480	.445	.243	55	30	.822	9.309	.329
490	.444	.241	55	41	.816	9.310	.330
500	.443	.239	55	53	.810	9.311	.331
510	.441	.237	56	04	.804	9.313	.332
520	.440	.235	56	15	.798	9.314	.333
530	.439	.233	56	26	.793	9.315	.334
540	.438	.231	56	37	.788	9.317	.335
550	.437	.229	56	49	.783	9.318	.336
560	.436	.227	57	00	.778	9.319	.338
570	.435	.225	57	11	.773	9.321	.339
580	.434	.223	57	22	.768	9.322	.340
590	.433	.221	57	33	.763	9.323	.341
600	.432	.219	57	44	.758	9.324	.342
610	.430	.218	57	56	.753	9.326	.343
620	.429	.216	58	07	.748	9.327	.344
630	.427	.214	58	18	.743	9.328	.345
640	.426	.212	58	29	.738	9.330	.347
650	.425	.210	58	40	.733	9.331	.348
660	.424	.208	58	51	.728	9.332	.349
670	.423	.206	59	02	.723	9.334	.351
680	.421	.204	59	13	.718	9.335	.352
690	.420	.202	59	24	.713	9.336	.353
700	.419	.201	59	34	.709	9.337	.354
710	.418	.199	59	45	.704	9.338	.355
720	.417	.197	59	55	.699	9.339	.356
730	.416	.195	60	06	.694	9.341	.358
740	.415	.193	60	16	.689	9.342	.359
750	.413	.192	60	27	.685	9.343	.360
760	.412	.190	60	37	.680	9.345	.361
670	.411	.188	60	48	.675	9.346	.363
780	.410	.186	60	58	.671	9.348	.364
790	.408	.184	61	09	.666	9.349	.365
800	.407	.182	61	19	.662	9.351	.366

$\varphi = 51°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.482	.300	51	29	.985	9.285	.311
110	.482	.299	51	33	.983	9.285	.311
120	.481	.298	51	36	.981	9.286	.311
130	.481	.297	51	39	.979	9.286	.312
140	.480	.296	51	43	.977	9.287	.312
150	.480	.295	51	47	.974	9.287	.313
160	.479	.294	51	52	.971	9.288	.313
170	.479	.293	51	56	.968	9.288	.314
180	.478	.292	52	01	.965	9.289	.314
190	.478	.291	52	06	.962	9.290	.315
200	.477	.290	52	11	.959	9.291	.315
210	.476	.289	52	17	.955	9.292	.316
220	.476	.287	52	23	.951	9.293	.317
230	.475	.286	52	29	.947	9.293	.317
240	.474	.285	52	37	.943	9.294	.318
250	.474	.283	52	45	.939	9.295	.319
260	.473	.282	52	53	.935	9.296	.319
270	.472	.280	53	01	.931	9.297	.320
280	.471	.279	53	10	.927	9.298	.321
290	.471	.277	53	18	.922	9.299	.321
300	.470	.275	53	27	.917	9.300	.322
310	.469	.273	53	36	.912	9.302	.323
320	.468	.271	53	45	.907	9.303	.324
330	.467	.270	53	54	.902	9.304	.325
340	.467	.268	54	04	.897	9.305	.326
350	.466	.266	54	13	.892	9.306	.327
360	.465	.264	54	23	.887	9.307	.328
370	.464	.262	54	32	.882	9.308	.329
380	.463	.260	54	41	.877	9.310	.330
390	.462	.258	54	51	.872	9.311	.331
400	.461	.256	55	02	.867	9.312	.332
410	.459	.254	55	13	.862	9.313	.334
420	.458	.252	55	24	.857	9.315	.335
430	.457	.250	55	34	.852	9.316	.336
440	.456	.249	55	44	.847	9.317	.337
450	.455	.247	55	54	.841	9.318	.338

$\varphi = 51°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.454	.245	56	05	.835	9.320	.339
470	.453	.243	56	16	.829	9.321	.340
480	.452	.241	56	27	.823	9.322	.341
490	.450	.239	56	38	.817	9.323	.342
500	.449	.237	56	50	.811	9.325	.343
510	.448	.235	57	01	.805	9.326	.345
520	.447	.233	57	12	.800	9.327	.346
530	.446	.231	57	22	.795	9.328	.347
540	.445	.229	57	33	.790	9.330	.348
550	.443	.227	57	45	.785	9.331	.349
560	.442	.225	57	56	.780	9.332	.350
570	.441	.223	58	07	.775	9.334	.351
580	.440	.221	58	17	.770	9.335	.352
590	.439	.219	58	28	.765	9.336	.353
600	.438	.217	58	39	.760	9.338	.354
610	.436	.216	58	51	.755	9.339	.356
620	.435	.214	59	02	.750	9.340	.357
630	.434	.212	59	12	.745	9.341	.358
640	.433	.210	59	23	.740	9.343	.359
650	.431	.208	59	34	.735	9.344	.361
660	.430	.206	59	45	.730	9.345	.362
670	.429	.204	59	56	.725	9.347	.363
680	.428	.202	60	07	.720	9.348	.365
690	.427	.200	60	17	.715	9.349	.366
700	.425	.199	60	21	.711	9.351	.367
710	.424	.197	60	38	.706	9.352	.368
720	.423	.195	60	48	.701	9.353	.370
730	.422	.193	60	58	.696	9.354	.371
740	.421	.191	61	08	.692	9.356	.372
750	.419	.189	61	19	.687	9.357	.373
760	.418	.187	61	29	.682	9.358	.375
770	.417	.185	61	40	.678	9.360	.376
780	.416	.184	61	50	.673	9.361	.377
790	.415	.182	62	00	.669	9.362	.378
800	.413	.180	62	10	.664	9.364	.379

$\varphi = 52°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.489	.298	52	27	.985	9.299	.322
110	.489	.297	52	30	.983	9.299	.322
120	.488	.296	52	34	.981	9.300	.323
130	.488	.295	52	38	.979	9.300	.323
140	.487	.294	52	42	.977	9.301	.324
150	.487	.293	52	46	.974	9.301	.324
160	.486	.292	52	51	.971	9.302	.325
170	.486	.291	52	55	.968	9.302	.325
180	.485	.290	53	00	.965	9.303	.326
190	.485	.289	53	05	.962	9.304	.326
200	.484	.288	53	10	.959	9.305	.327
210	.483	.286	53	16	.956	9.306	.327
220	.483	.285	53	22	.951	9.307	.328
230	.482	.284	53	29	.947	9.307	.329
240	.481	.282	53	36	.943	9.308	.329
250	.481	.281	53	44	.939	9.309	.330
260	.480	.279	53	52	.935	9.310	.331
270	.479	.278	54	00	.931	9.311	.332
280	.479	.276	54	09	.927	9.312	.332
290	.478	.274	54	17	.923	9.313	.333
300	.477	.273	54	26	.918	9.314	.334
310	.476	.271	54	35	.913	9.315	.335
320	.475	.269	54	43	.908	9.317	.336
330	.474	.267	54	52	.903	9.318	.337
340	.473	.265	55	02	.898	9.319	.338
350	.472	.264	55	11	.893	9.320	.339
360	.471	.262	55	21	.888	9.321	.340
370	.470	.260	55	30	.883	9.323	.341
380	.469	.258	55	39	.878	9.324	.342
390	.468	.256	55	49	.873	9.325	.344
400	.467	.254	55	59	.868	9.326	.345
410	.465	.253	56	10	.863	9.328	.346
420	.464	.251	56	21	.858	9.329	.347
430	.463	.249	56	31	.853	9.330	.348
440	.462	.247	56	41	.848	9.331	.349
450	.461	.245	56	51	.843	9.333	.350

$\varphi = 52^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.460	.243	57	02	.837	9.334	.351
470	.459	.241	57	13	.831	9.335	.352
480	.458	.239	57	24	.825	9.336	.354
490	.457	.237	57	35	.819	9.338	.355
500	.455	.235	57	46	.813	9.339	.356
510	.454	.233	57	57	.807	9.340	.357
520	.453	.231	58	08	.802	9.341	.358
530	.452	.229	58	18	.797	9.343	.359
540	.451	.227	58	29	.792	9.344	.360
550	.450	.225	58	40	.787	9.345	.362
560	.448	.223	58	51	.782	9.347	.363
570	.447	.221	59	02	.777	9.348	.364
580	.446	.219	59	12	.772	9.349	.365
590	.445	.217	59	22	.767	9.351	.366
600	.444	.215	59	33	.762	9.352	.367
610	.442	.214	59	45	.757	9.353	.369
620	.441	.212	59	56	.752	9.355	.370
630	.440	.210	60	06	.747	9.356	.371
640	.439	.208	60	16	.742	9.357	.372
650	.437	.206	60	27	.737	9.359	.373
660	.436	.204	60	38	.733	9.360	.375
670	.435	.202	60	49	.728	9.361	.376
680	.433	.201	61	00	.723	9.363	.377
690	.432	.199	61	10	.718	9.364	.378
700	.431	.197	61	20	.714	9.365	.380
710	.430	.195	61	30	.710	9.367	.381
720	.428	.193	61	40	.705	9.368	.382
730	.427	.191	61	50	.700	9.369	.383
740	.426	.190	62	00	.695	9.371	.385
750	.425	.188	62	10	.691	9.372	.386
760	.423	.186	62	20	.686	9.373	.387
770	.422	.184	62	31	.681	9.375	.389
780	.421	.182	62	41	.676	9.376	.390
790	.419	.180	62	51	.671	9.377	.391
800	.418	.178	63	01	.666	9.378	.392

$\varphi = 53°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{x}$	$\frac{y_0}{X}$
			°	′		—10	
100	.496	.295	53	26	.986	9.313	.334
110	.496	.294	53	29	.984	9.313	.334
120	.495	.293	53	33	.982	9.314	.335
130	.495	.292	53	37	.980	9.314	.335
140	.494	.291	53	41	.978	9.315	.336
150	.494	.290	53	45	.975	9.315	.336
160	.493	.289	53	50	.972	9.316	.337
170	.493	.288	53	54	.969	9.316	.337
180	.492	.287	53	59	.966	9.317	.338
190	.492	.285	54	05	.963	9.318	.338
200	.491	.284	54	10	.960	9.319	.339
210	.490	.283	54	15	.956	9.320	.339
220	.490	.282	54	21	.952	9.321	.340
230	.489	.281	54	28	.948	9.321	.341
240	.488	.279	54	35	.944	9.322	.341
250	.487	.287	54	43	.940	9.323	.342
260	.487	.276	54	51	.936	9.324	.343
270	.486	.275	54	59	.932	9.325	.343
280	.485	.273	55	08	.928	9.326	.344
290	.484	.271	55	16	.924	9.327	.345
300	.483	.270	55	25	.919	9.328	.346
310	.482	.268	55	34	.914	9.329	.347
320	.481	.266	55	42	.909	9.331	.348
330	.480	.264	55	51	.904	9.332	.349
340	.479	.262	56	00	.899	9.333	.350
350	.478	.261	56	09	.894	9.334	.351
360	.477	.259	56	19	.889	9.335	.352
370	.476	.257	56	28	.884	9.337	.353
380	.475	.255	56	37	.879	9.338	.354
390	.474	.253	56	47	.874	9.339	.356
400	.473	.251	56	56	.869	9.340	.357
410	.471	.249	57	07	.864	9.342	.358
420	.470	.248	57	18	.859	9.343	.359
430	.469	.246	57	28	.854	9.344	.360
440	.468	.244	57	38	.849	9.345	.361
450	.467	.242	57	48	.844	9.347	.363

$\varphi = 53^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		−10	
460	.466	.240	57	58	.838	9.348	.364
470	.465	.238	58	09	.832	9.349	.365
480	.464	.236	58	20	.826	9.350	.366
490	.462	.234	58	21	.820	9.352	.367
500	.461	.233	58	42	.814	9.353	.369
510	.460	.231	58	52	.809	9.354	.370
520	.459	.229	59	03	.804	9.355	.371
530	.458	.227	59	13	.779	9.357	.372
540	.457	.225	59	24	.794	9.358	.373
550	.456	.223	59	35	.789	9.359	.375
560	.455	.221	59	45	.784	9.360	.376
570	.453	.219	59	56	.779	9.362	.377
580	.452	.217	60	06	.774	9.363	.378
590	.451	.215	60	16	.769	9.364	.380
600	.450	.213	60	27	.764	9.366	.381
610	.448	.212	60	38	.759	9.367	.382
620	.447	.210	60	49	.754	9.368	.383
630	.446	.208	61	00	.749	9.369	.385
640	.444	.206	61	11	.744	9.371	.386
650	.443	.204	61	20	.739	9.372	.387
660	.442	.202	61	30	.735	9.373	.388
670	.440	.200	61	41	.730	9.375	.390
680	.439	.198	61	52	.725	9.376	.391
690	.438	.196	62	02	.720	9.377	.392
700	.436	.195	62	11	.715	9.379	.394
710	.435	.193	62	21	.711	9.380	.395
720	.434	.191	62	31	.706	9.381	.396
730	.432	.189	62	40	.701	9.382	.398
740	.431	.187	62	50	.696	9.384	.399
750	.430	.185	63	01	.692	9.385	.400
760	.428	.183	63	11	.687	9.386	.402
770	.427	.181	63	21	.682	9.388	.403
780	.426	.179	63	31	.677	9.389	.405
790	.424	.177	63	41	.673	9.390	.406
800	.423	.176	63	51	.668	9.392	.408

$\varphi = 54°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.502	.292	54	24	.986	9.327	.346
110	.501	.291	54	28	.984	9.327	.346
120	.501	.290	54	32	.982	9.328	.347
130	.501	.289	54	36	.980	9.328	.347
140	.500	.288	54	41	.978	9.329	.348
150	.500	.287	54	45	.975	9.329	.348
160	.499	.286	54	49	.972	9.330	.349
170	.499	.285	54	54	.969	9.330	.349
180	.498	.284	54	59	.966	9.331	.350
190	.498	.283	55	04	.963	9.332	.350
200	.497	.282	55	09	.960	9.333	.351
210	.496	.280	55	14	.956	9.333	.351
220	.496	.279	55	20	.952	9.334	.352
230	.495	.278	55	27	.948	9.335	.353
240	.494	.276	55	34	.944	9.336	.354
250	.493	.275	55	42	.940	9.337	.354
260	.493	.273	55	50	.936	9.338	.355
270	.492	.272	55	58	.932	9.339	.356
280	.491	.270	56	06	.928	9.340	.357
290	.491	.268	56	14	.924	9.341	.358
300	.489	.267	56	23	.920	9.342	.359
310	.488	.265	56	32	.915	9.344	.360
320	.487	.263	56	40	.910	9.345	.361
330	.486	.261	56	49	.905	9.346	.362
340	.485	.259	56	58	.900	9.347	.364
350	.484	.258	57	07	.895	9.348	.365
360	.483	.256	57	16	.890	9.350	.366
370	.482	.254	57	26	.885	9.351	.367
380	.481	.252	57	35	.880	9.352	.368
390	.480	.250	57	44	.875	9.353	.369
400	.479	.248	57	54	.870	9.354	.370
410	.477	.247	58	03	.865	9.356	.372
420	.476	.245	58	13	.860	9.357	.373
430	.475	.243	58	24	.855	9.358	.374
440	.474	.241	58	34	.850	9.359	.375
450	.473	.239	58	44	.845	9.361	.377

$\varphi = 54^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{V_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.471	.237	58	54	.840	9.362	.378
470	.470	.236	59	04	.834	9.363	.379
480	.469	.234	59	15	.828	9.364	.380
490	.468	.232	59	26	.822	9.366	.381
500	.467	.230	59	37	.816	9.367	.383
510	.465	.228	59	47	.811	9.368	.384
520	.464	.226	59	57	.806	9.369	.385
530	.463	.224	60	08	.801	9.371	.386
540	.462	.222	60	19	.796	9.372	.387
550	.461	.221	60	29	.791	9.373	.389
560	.459	.219	60	39	.786	9.374	.390
570	.458	.217	60	49	.781	9.376	.391
580	.457	.215	61	00	.776	9.377	.392
590	.456	.213	61	10	.771	9.378	.394
600	.455	.211	61	20	.766	9.380	.395
610	.453	.209	61	31	.761	9.381	.396
620	.452	.207	51	41	.756	9.382	.398
630	.451	.206	61	52	.751	9.383	.399
640	.449	.204	62	02	.745	9.385	.400
650	.448	.202	62	12	.741	9.386	.402
660	.447	.200	62	21	.736	9.387	.403
670	.445	.198	62	31	.731	9.389	.404
680	.444	.196	62	42	.726	9.390	.406
690	.443	.194	62	52	.721	9.391	.407
700	.441	.193	63	01	.717	9.393	.408
710	.439	.191	63	11	.712	9.394	.410
720	.438	.189	63	21	.707	9.395	.411
730	.437	.187	63	31	.702	9.396	.413
740	.436	.185	63	41	.698	9.398	.414
750	.434	.183	63	51	.693	9.399	.416
760	.433	.181	64	00	.688	9.400	.417
770	.432	.179	64	10	.684	9.402	.419
780	.430	.178	64	20	.679	9.403	.420
790	.429	.176	64	30	.674	9.404	.422
800	.428	.174	64	39	.674	9.406	.423

$\varphi = 55°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.508	.288	55	23	.987	9.342	.359
110	.508	.288	55	27	.985	9.342	.359
120	.507	.287	55	31	.983	9.343	.360
130	.507	.286	55	35	.981	9.343	.360
140	.506	.285	55	40	.979	9.344	.361
150	.506	.284	55	44	.976	9.344	.361
160	.505	.283	55	48	.973	9.345	.362
170	.505	.282	55	53	.970	9.346	.362
180	.504	.281	55	58	.967	9.346	.363
190	.504	.280	56	03	.964	9.347	.363
200	.503	.279	56	08	.961	9.348	.364
210	.502	.277	56	14	.957	9.348	.365
220	.501	.276	56	20	.953	9.349	.366
230	.501	.275	56	26	.949	9.350	.366
240	.500	.273	56	33	.945	9.351	.367
250	.499	.272	56	40	.941	9.352	.368
260	.498	.270	56	48	.937	9.353	.369
270	.497	.269	56	56	.933	9.354	.370
280	.497	.267	57	04	.929	9.355	.371
290	.496	.266	57	12	.925	9.356	.372
300	.495	264	37	21	.921	9.357	.373
310	.494	.263	57	29	.917	9.359	.374
320	.493	.261	57	38	.912	9.360	.375
330	.492	.259	57	46	.907	9.361	.376
340	.491	.257	57	55	.902	9.362	.378
350	.490	.255	58	04	.897	9.363	.379
360	.489	.253	58	13	.892	9.364	.380
370	.488	.251	58	22	.887	9.365	.381
380	.487	.249	58	31	.882	9.366	.382
390	.486	.247	58	40	.877	9.367	.383
400	.485	.245	58	50	.872	9.369	.384
410	.483	.244	58	59	.866	9.370	.386
420	.482	.242	59	09	.860	9.371	.387
430	.481	.240	59	19	.854	9.372	.388
440	.480	.238	59	29	.848	9.373	.389
450	.479	.236	59	39	.843	9.375	.391

$\varphi = 55^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.477	.234	59	49	.838	9.376	.392
470	.476	.233	59	59	.833	9.377	.393
480	.475	.231	60	10	.828	9.379	.394
490	.474	.299	60	20	.823	9.380	.396
500	.373	.227	60	31	.818	9.382	.397
510	.471	.225	60	41	.813	9.383	.398
520	.470	.224	60	51	.808	9.385	.399
530	.469	.222	61	02	.803	9.386	.401
540	.467	.220	61	12	.798	9.387	.402
550	.466	.218	61	22	.793	9.388	.403
560	.465	.216	61	32	.788	9.389	.405
570	.463	.214	61	42	.783	9.391	.406
580	.462	.212	61	52	.778	9.392	.407
590	.461	.210	62	02	.773	9.393	.409
600	.460	.208	62	12	.768	9.394	.410
610	.458	.207	62	23	.764	9.396	.411
620	.457	.205	62	33	.759	9.397	.313
630	.456	.203	62	43	.754	9.398	.414
640	.455	.201	62	53	.749	9.399	.415
650	.453	.200	63	03	.744	9.401	.417
660	.452	.198	63	12	.739	9.402	.418
670	.450	.196	63	22	.735	9.403	.419
680	.449	.194	63	32	.730	9.404	.421
690	.447	.192	63	42	.725	9.406	.422
700	.446	.190	63	51	.720	9.407	.424
710	.445	.189	64	01	.716	9.408	.425
720	.443	.187	64	11	.711	9.409	.426
730	.442	.185	64	21	.706	9.411	.428
740	.441	.183	64	30	.702	9.412	.429
750	.439	.181	64	40	.697	9.413	.431
760	.438	.180	64	49	.692	9.414	.433
770	.437	.178	64	59	.687	9.416	.434
780	.435	.176	65	08	.683	9.417	.435
790	.434	.174	65	18	.678	9.418	.437
800	.433	.172	65	27	.673	9.420	.439

$\varphi = 56^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{X}$	$\frac{y_0}{X}$
			°	′		−10	
100	.514	.284	56	22	.987	9.357	.373
110	.514	.283	56	25	.985	9.357	.373
120	.513	.283	56	29	.983	9.358	.374
130	.513	.282	56	33	.981	9.358	.374
140	.512	.281	56	38	.979	9.359	.375
150	.512	.280	56	43	.976	9.359	.375
160	.511	.279	56	48	.974	9.360	.376
170	.511	.278	56	52	.970	9.360	.376
180	.510	.277	56	57	.967	9.361	.377
190	.510	.276	57	02	.964	9.361	.378
200	.509	.275	57	07	.961	9.362	.378
210	.508	.273	57	12	.957	9.363	.379
220	.508	.272	57	18	.953	9.364	.379
230	.507	.271	57	25	.949	9.364	.380
240	.506	.269	57	32	.945	9.365	.381
250	.505	.268	57	39	.941	9.366	.382
260	.504	.266	57	47	.937	9.367	.383
270	.504	.265	57	55	.933	9.368	.384
280	.503	.263	58	02	.929	9.369	385
290	.502	.262	58	09	.925	9.370	.386
300	.501	.260	58	17	.921	9.371	.397
310	.500	.259	58	25	.917	9.372	.388
320	.499	.257	58	34	.913	9.374	.390
330	.498	.255	58	43	.908	9.375	.391
340	.497	.253	58	52	.903	9.376	.392
350	.496	.251	59	01	.898	9.377	.393
360	.495	.249	59	10	.893	9.378	.394
370	.494	.247	59	19	.888	9.380	.396
380	.493	.246	59	28	.883	9.381	.397
390	.492	.244	59	37	.878	9.382	.398
400	.491	.242	59	46	.873	9.383	.399
410	.489	.240	59	55	.867	9.385	.401
420	.488	.238	60	04	.861	9.386	.402
430	.487	.236	60	14	.855	9.387	.403
440	.486	.235	60	24	.849	9.388	.404
450	.485	.233	60	34	.844	9.390	.406

$\varphi = 56°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.484	.231	60	44	.839	9.391	.407
470	.482	.229	60	54	.834	9.392	.408
480	.481	.227	61	04	.829	9.393	.409
490	.480	.225	61	14	.824	9.395	.411
500	.479	.224	61	24	.819	9.396	.412
510	.477	.222	61	34	.814	9.397	.413
520	.476	.220	61	44	.809	9.398	.414
530	.475	.218	61	54	.804	9.400	.416
540	.473	.216	62	04	.799	9.401	.417
550	.472	.214	62	14	.794	9.402	.418
560	.471	.213	62	24	.789	9.403	.420
570	.469	.211	62	34	.784	9.405	.421
580	.468	.209	62	44	.779	9.406	.423
590	.466	.207	62	54	.774	9.407	.424
600	.465	.205	63	04	.769	9.408	.426
610	.464	.203	63	14	.765	9.410	.427
620	.462	.202	63	24	.760	9.411	.428
630	.461	.200	63	34	.755	9.412	.430
640	.459	.198	63	44	.750	9.413	.431
650	.458	.196	63	54	.746	9.415	.433
660	.456	.194	64	04	.741	9.416	.434
670	.455	.192	64	13	.737	9.417	.436
680	.453	.191	64	22	.732	9.418	.437
690	.452	.189	64	32	.727	9.420	.439
700	.451	.187	64	41	.722	9.421	.440
710	.449	.185	64	51	.717	9.422	.442
720	.448	.184	65	00	.713	9.423	.443
730	.447	.182	65	10	.708	9.425	.445
740	.445	.180	65	19	.703	9.426	.446
750	.444	.178	65	29	.699	9.427	.448
760	.443	.177	65	38	.694	9.428	.450
770	.442	.175	65	47	.689	9.430	.451
780	.440	.173	65	56	.685	9.431	.453
790	.439	.171	66	05	.680	9.432	.455
800	.438	.170	66	14	.675	9.434	.456

$\varphi = 57°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.520	.280	57	21	.987	9.372	.387
110	.520	.280	57	24	.985	9.372	.387
120	.519	.279	57	28	.983	9.373	.388
130	.519	.278	57	32	.980	9.373	.388
140	.518	.277	57	37	.978	9.374	.389
150	.518	.276	57	42	.976	9.374	.389
160	.517	.275	57	47	.973	9.375	.390
170	.517	.274	57	51	.970	9.375	.390
180	.516	.273	57	56	.967	9.376	.391
190	.516	.272	58	01	.964	9.376	.392
200	.515	.271	58	06	.961	9.377	.393
210	.514	.269	58	11	.958	9.378	.394
220	.514	.268	58	17	.954	9.379	.394
230	.513	.267	58	23	.950	9.379	.395
240	.512	.265	58	30	.946	9.380	.396
250	.511	.264	58	37	.942	9.381	.397
260	.510	.262	58	45	.938	9.382	.398
270	.510	.261	58	53	.934	9.383	.399
280	.509	.259	59	00	.930	9.384	.400
290	.508	.257	59	07	.926	9.385	.401
300	.507	.256	59	15	.922	9.386	.402
310	.506	.254	59	23	.918	9.387	.403
320	.505	.252	59	31	.914	9.388	.404
330	.504	.250	59	39	.909	9.389	.406
340	.503	.249	59	48	.904	9.391	.407
350	.502	.247	59	57	.899	9.392	.408
360	.501	.245	60	05	.894	9.393	.409
370	.500	.243	60	14	.889	9.394	.411
380	.499	.242	60	23	.884	9.395	.412
390	.498	.240	60	32	.879	9.396	.413
400	.497	.238	60	41	.874	9.397	.414
410	.495	.236	60	50	.868	9.399	.416
420	.494	.235	60	59	.862	9.400	.417
430	.493	.233	61	08	.856	9.401	.418
440	.492	.231	61	18	.851	9.402	.420
450	.490	.230	61	28	.846	9.404	.421

$\varphi = 57^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω	$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			° ′		—10	
460	.489	.228	61 37	.841	9.405	.422
470	.488	.226	61 47	.836	9.406	.424
480	.487	.225	61 57	.831	9.408	.425
490	.485	.223	62 07	.826	9.409	.426
500	.484	.221	62 17	.821	9.410	.428
510	.483	.219	62 26	.816	9.412	.429
520	.481	.218	62 36	.811	9.413	.430
530	.480	.216	62 46	.806	9.414	.432
540	.479	.214	62 56	.801	9.415	.433
550	.477	.212	63 06	.796	9.416	.434
560	.476	.210	63 15	.791	9.418	.436
570	.475	.208	63 25	.786	9.419	.437
580	.473	.206	63 35	.781	9.420	.439
590	.472	.204	60 45	.776	9.421	.440
600	.470	.202	63 55	.771	9.422	.442
610	.469	.201	64 04	.767	9.424	.443
620	.468	.199	64 14	.762	9.425	.445
630	.466	.197	64 24	.757	9.426	.446
640	.465	.195	64 34	.752	9.427	.448
650	.463	.193	64 44	.748	9.429	.449
660	.462	.191	64 53	.743	9.430	.451
670	.460	.190	65 02	.738	9.431	.452
680	.459	.188	65 11	.734	9.432	.454
690	.457	.186	65 21	.729	9.434	.455
700	.456	.184	65 30	.724	9.435	.457
710	.555	.183	65 39	.719	9.436	.458
720	.453	.181	65 48	.715	9.437	.460
730	.452	.179	65 58	.710	9.439	.462
740	.451	.177	66 07	.705	9.440	.463
750	.450	.176	66 17	701	9.441	.465
760	.448	.174	66 26	.696	9.443	.467
770	.447	.172	66 35	.691	9.444	.468
780	.446	.170	66 43	.687	9.445	.470
790	.445	.169	66 52	.682	9.446	.472
800	.443	.167	67 00	.677	9.448	.474

$\varphi = 58°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.526	.275	58	21	.988	9.387	.402
110	.526	.275	58	25	.986	9.387	.402
120	.525	.274	58	29	.984	9.388	.403
130	.525	.273	58	33	.981	9.388	.404
140	.524	.273	58	37	.979	9.389	.404
150	.524	.272	58	41	.977	9.389	.405
160	.523	.271	58	45	.974	9.390	.406
170	.522	.270	58	50	.971	9.390	.406
180	.522	.269	58	55	.968	9.391	.407
190	.521	.268	59	00	.965	9.391	.408
200	.521	.267	59	05	.962	9.392	.409
210	.520	.266	59	11	.959	9.393	.410
220	.520	.264	59	17	.955	9.394	.410
230	.519	.263	59	23	.951	9.394	.411
240	.518	.261	59	29	.947	9.395	.412
250	.517	.260	59	35	.943	9.396	.413
260	.516	.258	59	42	.939	9.397	.414
270	.516	.257	59	49	.935	9.398	.415
280	.515	.255	59	56	.931	9.399	.416
290	.514	.254	60	04	.927	9.400	.417
300	.513	.252	60	12	.923	9.401	.418
310	.512	.250	60	22	.919	9.402	.419
320	.511	.249	60	28	.915	9.403	.421
330	.510	.247	60	36	.910	9.404	.422
340	.509	.245	60	44	.905	9.406	.423
350	.508	.243	60	53	.900	9.407	.424
360	.507	.241	61	02	.895	9.408	.425
370	.506	.240	61	10	.890	9.409	.427
380	.505	.238	61	19	.885	9.410	.428
390	.504	.236	61	28	.880	9.411	.429
400	.502	.234	61	37	.875	9.412	.430
410	.501	.233	61	36	.869	9.414	.432
420	.500	.231	61	55	.863	9.415	.433
430	.499	.229	62	04	.858	9.416	.434
440	.497	.227	62	13	.853	9.417	.436
450	.496	.226	62	22	.848	9.419	.437

$\varphi = 58°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.495	.224	62	32	.843	9.420	.439
470	.493	.222	62	41	.838	9.421	.440
480	.492	.220	62	51	.833	9.423	.442
490	.491	.219	63	00	.828	9.424	.443
500	.489	.217	63	10	.823	9.425	.445
510	.488	.215	63	20	.818	9.427	.446
520	.487	.213	63	29	.813	9.428	.447
530	.485	.211	63	39	.808	9.429	.449
540	.484	.210	63	48	.803	9.430	.450
550	.483	.208	63	58	.798	9.431	.451
560	.481	.206	64	08	.793	9.433	.453
570	.480	.204	64	17	.788	9.434	.454
580	.478	.202	64	26	.783	9.435	.456
590	.477	.201	64	36	.778	9.436	.457
600	.475	.199	64	45	.773	9.437	.459
610	.474	.197	64	55	.769	9.439	.460
620	.473	.195	65	05	.764	9.440	.462
630	.471	.193	65	15	.759	9.441	.463
640	.470	.192	65	24	.754	9.442	.465
650	.469	.190	65	33	.750	9.444	.466
660	.467	.188	65	43	.745	9.445	.468
670	.466	.186	65	52	.740	9.446	.469
680	.464	.184	66	01	.736	9.447	.471
690	.463	.183	66	10	.731	9.449	.473
700	.461	.181	66	19	.726	9.450	.475
710	.460	.179	66	28	.721	9.451	.476
720	.459	.177	66	37	.717	9.452	.478
730	.457	.175	66	46	.712	9.454	.480
740	.456	.174	66	55	.707	9.455	.481
750	.455	.172	67	04	.703	9.456	.483
760	.453	.170	67	13	.698	9.458	.485
770	.452	.169	67	22	.693	9.459	.486
780	.451	.167	67	30	.689	9.460	.488
790	.450	.165	67	38	.684	9.461	.490
800	.448	.164	67	46	.679	9.463	.492

$\varphi = 59°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.532	.270	59	20	.988	9.402	.418
110	.532	.270	59	24	.986	9.402	.418
120	.531	.269	59	28	.984	9.403	.419
130	.531	.268	59	32	.982	9.403	.420
140	.530	.268	59	36	.980	9.404	.420
150	.530	.267	59	40	.977	9.404	.421
160	.529	.266	59	44	.974	9.405	.422
170	.529	.265	59	48	.971	9.405	.423
180	.528	.264	59	53	.968	9.406	.423
190	.528	.263	59	58	.965	9.406	.424
200	.527	.262	60	03	.962	9.407	.425
210	.526	.261	60	08	.959	9.408	.426
220	.526	.259	60	14	.956	9.408	.427
230	.525	.258	60	20	.952	9.409	.428
240	.524	.257	60	26	.948	9.410	.429
250	.523	.255	60	33	.944	9.411	.430
260	.522	.254	60	40	.940	9.412	.431
270	.521	.253	60	47	.936	9.413	.432
280	.520	.251	60	54	.932	9.414	.433
290	.519	.250	61	01	.928	9.415	.434
300	.518	.247	61	09	.924	9.416	.435
310	.517	.246	61	17	.920	9.417	.436
320	.516	.245	61	25	.916	9.418	.438
330	.515	.243	61	33	.911	9.419	.439
340	.514	.241	61	41	.906	9.421	.440
350	.513	.239	61	49	.901	9.422	.441
360	.511	.238	61	57	.896	9.423	.442
370	.510	.236	62	06	.891	9.424	.444
380	.509	.234	62	14	.886	9.425	.445
390	.508	.232	62	23	.881	9.426	.446
400	.507	.230	62	31	.876	9.427	.447
410	.505	.229	62	40	.870	9.429	.449
320	.504	.227	62	49	.865	9.430	.450
430	.503	.225	62	58	.860	9.431	.451
440	.502	.223	63	07	.855	9.432	.453
450	.500	.222	63	16	.850	9.434	.454

$\varphi = 59°$

$\frac{V}{C}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.499	.220	63	25	.845	9.435	.456
470	.498	.218	63	34	.840	9.436	.457
480	.496	.216	63	43	.835	9.438	.459
490	.495	.214	63	52	.830	9.439	.460
500	.494	.213	64	02	.825	9.440	.462
510	.492	.211	64	12	.820	9.442	.463
520	.491	.209	64	21	.815	9.443	.465
530	.490	.207	64	30	.810	9.444	.466
540	.488	.205	64	39	.805	9.445	.468
550	.487	.204	64	49	.800	9.446	.469
560	.486	.202	64	58	.795	9.448	.471
570	.484	.200	65	07	.790	9.449	.472
580	.483	.198	65	17	.785	9.450	.474
590	.482	.197	65	27	.780	9.451	.475
600	.480	.195	65	36	.775	9.452	.477
610	.479	.193	65	46	.771	9.454	.479
620	.478	.191	65	55	.766	9.455	.481
630	.476	.190	66	04	.761	9.456	.482
640	.475	.188	66	13	.756	9.457	.484
650	.473	.186	66	22	.752	9.459	.486
660	.472	.185	66	31	.747	9.460	.488
670	.470	.183	66	40	.742	9.461	.489
680	.469	.181	66	49	.738	9.462	.491
690	.467	.179	66	58	.733	9.464	.493
700	.466	.178	67	07	.728	9.465	.494
710	.464	.176	67	16	.723	9.466	.496
720	.463	.174	67	25	.719	9.467	.498
730	.461	.173	67	34	.714	9.469	.500
740	.460	.171	67	42	.709	9.470	.501
750	.459	.169	67	50	.705	9.471	.503
760	.457	.167	67	58	.700	9.473	.505
770	.456	.166	68	07	.695	9.474	.506
780	.455	.164	68	15	.691	9.475	.508
790	.453	.162	68	23	.686	9.476	.510
800	.452	.161	68	31	.681	9.478	.511

$\varphi = 60°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.537	.265	60	19	.989	9.417	.435
110	.537	.264	60	23	.987	9.417	.435
120	.536	.264	60	27	.985	9.418	.436
130	.536	.263	60	31	.983	9.418	.437
140	.535	.263	60	35	.981	9.419	.437
150	.535	.262	60	39	.978	9.419	.438
160	.534	.261	60	43	.975	9.420	.439
170	.533	.260	60	47	.972	9.421	.439
180	.532	.259	60	52	.969	9.421	.440
190	.532	.258	60	57	.966	9.422	.441
200	.531	.257	61	02	.963	9.423	.442
210	.530	.255	61	05	.960	9.243	.443
220	.530	.254	61	13	.957	9.424	.444
230	.529	.253	61	19	.953	9.424	.445
240	.528	.252	61	25	.949	9.425	.446
250	.527	.250	61	31	.945	9.426	.447
260	.526	.249	61	38	.941	9.427	.448
270	.526	.248	61	44	.937	9.428	.449
280	.525	.246	61	51	.933	9.429	.450
290	.524	.245	61	58	.929	9.430	.451
300	.523	.243	62	05	.925	9.431	.452
310	.522	.242	62	13	.921	9.432	.454
320	.521	.240	62	20	.917	9.433	.455
330	.520	.238	62	28	.913	9.434	.456
340	.519	.236	62	36	.908	9.435	.457
350	.518	.235	62	44	.903	9.437	.459
360	.516	.233	62	52	.898	9.438	.460
370	.515	.231	63	01	.893	9.439	.461
380	.514	.229	63	09	.888	9.440	.463
390	.513	.227	63	18	.883	9.441	.464
400	.512	.225	63	26	.878	9.442	.465
410	.510	.224	63	35	.873	9.444	.467
420	.509	.222	63	43	.868	9.445	.468
430	.508	.220	63	52	.863	9.446	.469
440	.506	.218	64	00	.858	9.447	.471
450	.505	.217	64	09	.852	9.449	.472

$\varphi = 60^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.504	.215	64	18	.847	9.450	.474
470	.503	.213	64	27	.842	9.451	.475
480	.501	.212	64	36	.837	9.452	.477
490	.500	.210	64	45	.832	9.454	.478
500	.499	.209	64	54	.827	9.455	.480
510	.497	.207	65	03	.822	9.456	.481
520	.496	.205	65	12	.817	9.457	.483
530	.494	.203	65	21	.812	9.458	.485
540	.493	.201	65	30	.807	9.459	.486
550	.492	.199	65	39	.802	9.460	.488
560	.490	.197	65	48	.797	9.462	.490
570	.489	.196	65	57	.792	9.463	.491
580	.488	.194	66	07	.787	9.464	.493
590	.486	.192	66	16	.782	9.465	.495
600	.485	.191	66	25	.777	9.467	.496
610	.484	.189	66	34	.773	9.468	.498
620	.482	.187	66	43	.768	9.469	.500
630	.481	.185	66	52	.763	9.471	.501
640	.479	.184	67	01	.758	9.472	.503
650	.478	.182	67	10	.754	9.473	.505
660	.476	.180	67	19	.749	9.474	.507
670	.474	.178	67	28	.744	9.476	.508
680	.473	.177	67	37	.740	9.477	.510
690	.471	.175	67	46	.735	9.478	.512
700	.470	.174	67	54	.730	9.480	.513
710	.468	.172	68	03	.726	9.481	.515
720	.467	.170	68	11	.721	9.482	.517
730	.465	.168	68	20	.716	9.483	.519
740	.464	.167	68	28	.712	9.485	.520
750	.462	.165	68	36	.707	9.486	.522
760	.461	.163	68	44	.702	9.487	.524
770	.459	.161	68	52	.698	9.489	.525
780	.458	.160	69	00	.693	9.490	.527
790	.457	.159	69	08	.689	9.491	.529
800	.456	.158	69	15	.686	9.493	.531

$\varphi = 61^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	'		—10	
100	.542	.260	61	18	.989	9.432	.453
110	.542	.260	61	22	.987	9.432	.453
120	.541	.259	61	26	.985	9.433	.454
130	.541	.258	61	30	.983	9.433	.455
140	.540	.258	61	34	.980	9.434	.455
150	.540	.257	61	38	.978	9.434	.456
160	.539	.256	61	62	.975	9.435	.457
170	.539	.255	61	46	.972	9.436	.457
180	.538	.254	61	50	.967	9.436	.458
190	.538	.253	61	55	.966	9.437	.459
200	.537	.252	62	00	.963	9.438	.460
210	.536	.250	62	05	.960	9.438	.461
220	.535	.249	62	11	.957	9.439	.462
230	.535	.248	62	17	.953	9.440	.463
240	.534	.247	62	23	.949	9.440	.464
250	.533	.245	62	29	.945	9.441	.465
260	.532	.244	62	35	.941	9.442	.466
270	.531	.243	62	41	.937	9.443	.467
280	.530	.241	62	48	.933	9.444	.468
290	.529	.240	62	55	.929	9.445	.469
300	.528	.238	63	02	.925	9.446	.470
310	.527	.236	63	09	.921	9.447	.472
320	.526	.235	63	16	.917	9.448	.473
330	.525	.233	63	23	.913	9.449	.474
340	.524	.231	63	31	.909	9.450	.475
350	.523	.229	63	39	.904	9.452	.477
360	.521	.228	63	47	.899	9.453	.478
370	.520	.226	63	55	.894	9.454	.479
380	.519	.224	64	04	.889	9.455	.481
390	.518	.222	64	12	.884	9.456	.482
400	.517	.220	64	20	.879	9.457	.484
410	.515	.219	64	29	.874	9.459	.485
420	.514	.217	64	37	.869	9.460	.487
430	.513	.215	64	45	.864	9.461	.489
440	.512	.213	64	54	.859	9.462	.490
450	.510	.212	65	02	.853	9.463	.492

$\varphi = 61°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		−10	
460	.509	.210	65	11	.848	9.465	.494
470	.508	.208	65	20	.843	9.466	.495
480	.506	.207	65	28	.838	9.467	.497
490	.505	.205	65	37	.833	9.468	.498
500	.504	.204	65	45	.828	9.470	.500
510	.502	.202	65	54	.823	9.471	.501
520	.501	.200	66	03	.818	9.472	.503
530	.500	.199	66	11	.813	9.473	.505
540	.498	.197	66	20	.808	9.475	.506
550	.497	.195	66	29	.803	9.476	.508
560	.496	.194	66	38	.798	9.477	.510
570	.494	.192	66	47	.793	9.478	.511
580	.493	.190	66	56	.788	9.480	.513
590	.491	.188	67	05	.783	9.481	.515
600	.490	.187	67	14	.778	9.482	.516
610	.488	.185	67	23	.774	9.483	.518
620	.487	.184	67	32	.769	9.485	.520
630	.485	.182	67	41	.764	9.486	.521
640	.484	.180	67	49	.760	9.487	.523
650	.482	.179	67	58	.755	9.488	.525
660	.481	.177	68	07	.750	9.490	.526
670	.479	.175	68	15	.746	9.491	.528
680	.478	.174	68	24	.741	9.492	.530
690	.476	.172	68	32	.737	9.493	.532
700	.475	.170	68	40	.732	9.495	.533
710	.473	.169	68	49	.728	9.496	.535
720	.472	.167	68	57	.723	9.497	.537
730	.470	.165	69	05	.718	9.498	.539
740	.469	.164	69	13	.714	9.500	.541
750	.467	.162	69	21	.709	9.501	.542
760	.466	.161	69	29	.704	9.502	.544
770	.464	.159	69	37	.700	9.504	.546
780	.463	.158	69	45	.695	9.505	548
790	.462	.156	69	52	.691	9.506	.550
800	.461	.155	69	59	.686	9.508	.552

$\varphi = 62^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{V_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		−10	
100	547	.254	62	17	.989	9.448	.472
110	.547	.254	62	21	.987	9.448	.472
120	.546	.253	62	25	.985	9.449	.473
130	546	252	62	29	.983	9.449	.474
140	.545	.252	62	33	.981	9.450	.475
150	.545	.251	62	37	.978	9.450	.476
160	.544	.250	62	41	.975	9.451	.476
170	.544	.249	62	45	.972	9.452	.477
180	.543	.248	62	49	.969	9.452	.478
190	.543	.247	62	54	.966	9.453	.479
200	.542	.246	62	59	.963	9.454	.480
210	.541	.244	63	04	.960	9.454	.481
220	.540	.243	63	09	.957	9.455	.482
230	.540	.242	63	15	.954	9.456	.483
240	.539	.241	63	21	.950	9.456	.484
250	.538	.239	63	27	.946	9.457	.485
260	.537	.238	63	33	.942	9.458	.486
270	.536	.237	63	39	.938	9.459	.487
280	.535	.235	63	45	.943	9.460	.488
290	.534	.234	63	51	.930	9.461	.489
300	.533	.232	63	58	.926	9.462	.490
310	.532	.231	64	05	.922	9.463	.492
320	.531	.229	64	12	.918	9.464	.493
330	.530	.227	64	19	.914	9.465	.494
340	.529	.226	64	26	.910	9.466	.495
350	.528	.224	64	34	.905	9.468	.497
360	.526	.222	64	42	.900	9.469	.498
370	.525	.220	64	50	.895	9.470	.499
380	.524	.219	64	58	.890	9.471	.501
390	.523	.217	65	06	.885	9.472	.502
400	.522	.215	65	14	.880	9.473	.504
410	.520	.214	65	23	.875	9.475	.506
420	.519	.212	65	31	.870	9.476	.507
430	.518	.210	65	39	.865	9.477	.509
440	.517	.209	65	47	.860	9.478	.511
450	.515	.207	65	55	.855	9.479	.512

$\varphi = 62°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.514	.206	66	03	.850	9.481	.514
470	.513	.204	66	12	.845	9.482	.516
480	.512	.203	66	20	.840	9.483	.517
490	.510	.201	66	28	.835	9.484	.519
500	.509	.200	66	36	.830	9.486	.521
510	.508	.198	66	45	.825	9.487	.522
520	.506	.196	66	53	.820	9.488	.524
530	.505	.195	67	01	.815	9.489	.526
540	.503	.193	67	10	.810	9.491	.528
550	.502	.191	67	18	.805	9.492	.729
560	.500	.190	67	27	.800	9.493	.531
570	.499	.188	67	35	.795	9.494	.533
580	.497	.186	67	44	.790	9.496	.535
590	.496	.185	67	53	.785	9.497	.536
600	.494	.183	68	02	.780	9.498	.538
610	.493	.181	68	11	.776	9.499	.540
620	.491	.180	68	20	.771	9.501	.541
630	.490	.178	68	29	.766	9.502	.543
640	.488	.176	68	37	.762	9.503	.545
650	.487	.175	68	45	.757	9.504	.546
660	.485	.173	68	53	.752	9.506	.548
670	.484	.171	69	01	.748	9.507	.550
680	.482	.170	69	10	.743	9.508	.551
690	.481	.168	69	18	.739	9.509	.553
700	.479	.166	69	26	.734	9.511	.555
710	.478	.165	69	34	.730	9.512	.557
720	.476	.163	69	42	.725	9.513	.559
730	.475	.162	69	50	.720	9.514	.561
740	.473	.160	69	58	.716	9.516	.563
750	.472	.159	70	05	.711	9.517	.565
760	.470	.157	70	13	.706	9.518	.567
770	.469	.156	70	20	.702	9.520	.569
780	.467	.155	70	28	.697	9.521	.571
790	.466	.153	70	35	.693	9.522	.573
800	.464	.152	70	42	.688	9.524	.575

$\varphi = 63^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{X}$	$\frac{y_0}{X}$
			°	'		—10	
100	.552	.248	63	15	.990	9.464	.492
110	.552	.248	63	19	.988	9.464	.493
120	.551	.247	63	23	.986	9.465	.494
130	.551	.246	63	27	.984	9.465	.495
140	.550	.245	63	31	.982	9.466	.495
150	.550	.245	63	35	.979	9.466	.496
160	.549	.244	63	39	.976	9.467	.497
170	.549	.243	63	43	.973	9.467	.498
180	.548	.242	63	47	.970	9.468	.499
190	.548	.241	63	52	.967	9.469	.500
200	.547	.240	63	57	.964	9.470	.501
210	.546	.238	64	02	.961	9.470	.502
220	.545	.237	64	07	.958	9.471	.503
230	.544	.236	64	12	.955	9.472	.504
240	.544	.235	64	18	.951	9.472	.505
250	.543	.233	64	24	.947	9.473	.506
260	.542	.232	64	30	.943	9.474	.507
270	.541	.231	64	36	.939	9.475	.508
280	.540	.229	64	42	.935	9.476	.509
290	.539	.228	64	48	.931	9.477	.510
300	.538	.227	64	54	.927	9.478	.511
310	.537	.225	65	01	.923	9.479	.513
320	.536	.224	65	08	.919	9.480	.514
330	.535	.222	65	15	.915	9.481	.515
340	.534	.220	65	22	.911	9.482	.517
350	.533	.219	65	29	.906	9.484	.518
360	.531	.217	65	37	.901	5.485	.520
370	.530	.215	65	44	.896	9.486	.521
380	.529	.214	65	52	.891	9.487	.523
390	.528	.212	66	00	.886	9.488	.524
400	.527	.210	66	08	.881	9.489	.526
410	.525	.209	66	16	.876	9.491	.527
420	.524	.207	66	23	.871	9.492	.529
430	.523	.206	66	31	.866	9.493	.531
440	.522	.204	66	39	.861	9.494	.532
450	.520	.203	66	47	.857	9.495	.534

$\varphi = 63°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.519	.201	66	55	.852	9.497	.536
470	.518	.200	67	03	.847	9.498	.537
480	.516	.198	67	11	.842	9.499	.539
490	.515	.197	67	19	.837	9.500	.541
500	.513	.195	67	28	.832	9.502	.543
510	.512	.193	67	36	.827	9.503	.544
520	.510	.192	67	44	.822	9.504	.546
530	.509	.190	67	52	.817	9.505	.548
540	.507	.189	68	00	.812	9.507	.550
550	.506	.187	68	08	.807	9.508	.552
560	.505	.186	68	17	.802	9.509	.553
570	.503	.184	68	25	.797	9.510	.555
580	.501	.183	68	33	.792	9.512	.557
390	.500	.171	68	41	.787	9.513	.559
600	.498	.179	68	49	.782	9.514	.561
610	.497	.178	68	58	.777	9.515	.562
620	.495	.176	69	06	.773	9.517	.564
630	.494	.174	69	15	.768	9.518	.566
640	.492	.172	69	24	.764	9.519	.568
650	.491	.171	69	32	.759	9.520	.570
660	.489	.169	69	40	.754	9.522	.571
670	.488	.167	69	48	.750	9.523	.573
680	.486	.166	69	56	.745	9.524	.575
690	.485	.164	70	04	.741	9.525	.577
700	.483	.162	70	12	.736	9.527	.579
710	.482	.161	70	20	.732	9.528	.581
720	.480	.159	70	27	.727	9.529	.583
730	.479	.158	70	34	.722	9.530	.585
740	.477	.156	70	42	.718	9.532	.587
750	.476	.155	70	49	.713	9.533	.589
760	.474	.153	70	56	.708	9.534	.591
770	.473	.152	71	04	.704	9.536	.593
780	.471	.150	71	11	.699	9.537	.595
790	.470	.149	71	18	.695	9.538	.597
800	.468	.148	71	25	.690	9.540	.599

$\varphi = 64°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.557	.241	64	14	.990	9.481	.514
110	.557	.241	64	18	.988	9.481	.515
120	.556	.241	64	22	.986	9.482	.516
130	.556	.240	64	26	.984	9.482	.517
140	.555	.240	64	30	.982	9.483	.517
150	.555	.239	64	34	.979	9.483	.518
160	.554	.238	64	38	.976	9.484	.519
170	.554	.237	64	42	.973	9.484	.520
180	.553	.236	64	46	.970	9.485	.521
190	.553	.235	64	50	.967	9.486	.522
200	.552	.234	64	54	.964	9.487	.523
210	.551	.232	64	59	.961	9.487	.524
220	.550	.231	65	04	.958	9.488	.525
230	.549	.230	65	09	.955	9.489	.526
240	.549	.229	65	14	.952	9.489	.527
250	.548	.227	65	20	.948	9.490	.528
260	.547	.226	65	26	.944	9.491	.529
270	.546	.225	65	32	.940	9.492	.530
280	.545	.223	65	38	.936	9.493	.531
290	.544	.222	65	44	.932	9.494	.533
300	.543	.221	65	50	.928	9.495	.534
310	.541	.219	65	57	.924	9.496	.535
320	.540	.218	66	04	.920	9.497	.537
530	.539	.216	66	11	.916	9.498	.538
340	.538	.215	66	18	.912	9.499	.540
350	.537	.213	66	25	.907	9.500	.541
360	.535	.212	66	32	.902	9.501	.543
370	.534	.210	66	39	.897	9.502	.544
380	.533	.209	66	47	.892	9.503	.546
390	.532	.207	66	54	.887	9.504	.547
400	.531	.205	67	02	.882	9.505	.549
410	.529	.204	67	09	.878	9.507	.551
420	.528	.202	67	17	.873	9.508	.552
430	.527	.201	67	24	.868	9.509	.554
440	.526	.199	67	32	.863	9.510	.556
450	.524	.198	67	40	.858	9.511	.558

$\varphi = 64°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.523	.196	67	47	.853	9.513	.560
470	.522	.193	67	55	.848	9.514	.563
480	.520	.193	68	03	.843	9.515	.563
490	.519	.192	68	11	.838	9.516	.565
500	.517	.190	68	19	.834	9.518	.567
510	.516	.188	68	27	.829	9.519	.569
520	.514	.187	68	35	.824	9.520	.571
530	.513	.185	68	43	.819	9.521	.572
540	.511	.184	68	51	.814	9.523	.574
550	.510	.182	68	59	.809	9.524	.576
560	.508	.180	69	06	.804	9.525	.578
570	.507	.179	69	14	.799	9.526	.580
580	.505	.177	69	22	.794	9.528	.582
590	.504	.175	69	30	.789	9.529	.584
600	.502	.174	69	38	.784	9.530	.586
610	.501	.172	69	45	.779	9.531	.587
620	.499	.170	69	53	.775	9.533	.589
630	.498	.169	70	01	.770	9.534	.591
640	.496	.167	70	09	.766	9.535	.593
650	.495	.165	70	16	.761	9.536	.595
660	.493	.164	70	24	.756	9.538	.597
670	.492	.162	70	32	.752	9.539	.599
680	.490	.161	70	39	.747	9.540	.601
690	.488	.159	70	47	.742	9.541	.603
700	.487	.158	70	54	.738	9.543	.605
710	.485	.156	71	02	.733	9.544	.607
720	.484	.155	71	08	.729	9.545	.609
730	.482	.153	71	17	.724	9.546	.611
740	.481	.152	71	24	.720	9.548	.613
750	.479	.150	71	31	.715	9.549	.615
760	.478	.149	71	38	.711	9.550	.617
770	.476	.148	71	46	.706	9.551	.619
780	.475	.146	71	53	.701	9.553	.621
790	.473	.145	72	00	.697	9.554	.623
800	.472	.144	72	07	.692	9.556	.625

$\varphi = 65°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.562	.234	65	12	.991	9.498	.538
110	.562	.234	65	16	.989	9.498	.538
120	.561	.234	65	20	.987	9.499	.539
130	.561	.233	65	24	.985	9.500	.540
140	.560	.233	65	28	.983	9.500	.541
150	.560	.232	65	32	.980	9.501	.542
160	.559	.231	65	36	.977	9.501	.543
170	.559	.230	65	40	.974	9.502	.544
180	.558	.229	65	44	.971	9.502	.545
190	.558	.228	65	48	.968	9.503	.546
200	.557	.227	65	53	.965	9.504	.547
210	.556	.226	65	57	.962	9.504	.548
220	.555	.225	66	02	.959	9.505	.549
230	.554	.223	66	07	.956	9.506	.550
240	.553	.222	66	12	.953	9.506	.552
250	.552	.221	66	17	.949	9.507	.553
260	.551	.220	66	23	.945	9.508	.554
270	.550	.219	66	29	.941	9.509	.555
280	.549	.217	66	35	.937	9.510	.557
290	.548	.216	66	41	.933	9.511	.558
300	.547	.215	66	47	.929	9.512	.559
310	.545	.213	66	53	.925	9.513	.561
320	.544	.212	66	59	.921	9.514	.562
330	.543	.210	67	06	.917	9.515	.563
340	.542	.209	67	13	.913	9.516	.565
350	.541	.207	67	20	.909	9.517	.566
360	.539	.206	67	27	.904	9.518	.568
370	.538	.204	67	34	.899	9.519	.569
380	.537	.203	67	41	.894	9.520	.571
390	.536	.201	67	48	.889	9.521	.572
400	.535	.200	67	55	.884	9.522	.574
410	.533	.198	68	02	.879	9.523	.756
420	.532	.197	68	09	.874	9.524	.577
430	.531	.195	68	16	.869	9.525	.579
440	.530	.194	68	24	.864	9.526	.581
450	.528	.192	68	32	.859	9.528	.583

$\varphi = 65°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.527	.191	68	39	.854	9.529	.585
470	.526	.189	68	47	.849	9.530	.587
480	.524	.188	68	54	.844	9.531	.588
490	.523	.186	69	02	.839	9.532	.590
500	.521	.185	69	10	.835	9.534	.592
510	.520	.183	69	18	.830	9.535	.594
520	.518	.181	69	25	.825	9.536	.596
530	.517	.180	69	33	.820	9.537	.598
540	.515	.178	69	40	.815	9.538	.600
550	.514	.177	69	48	.810	9.538	.602
560	.512	.175	69	55	.805	9.540	.604
570	.511	.174	70	03	.800	9.542	.606
580	.509	.172	70	10	.795	9.543	.608
590	.508	.171	70	18	.790	9.544	.610
600	.506	.169	70	25	.786	9.545	.612
610	.505	.168	70	32	.781	9.547	.614
620	.503	.166	70	39	.776	9.548	.616
630	.502	.165	70	47	.771	9.549	.618
640	.500	.163	70	54	.767	9.550	.620
650	.499	.162	71	02	.762	9.552	.622
660	.497	.160	71	09	.758	9.553	.624
670	.496	.159	71	16	.753	9.555	.626
680	.494	.157	71	23	.749	9.556	.628
690	.493	.155	71	30	.744	9.557	.630
700	.491	.154	71	37	.740	9.559	.632
710	.490	.152	71	44	.735	9.560	.635
720	.488	.151	71	52	.731	9.561	.637
730	.487	.149	71	59	.726	9.563	.639
740	.485	.148	72	06	.722	9.564	.641
750	.484	.146	72	13	.717	9.565	.643
760	.482	.145	72	20	.713	9.567	.645
770	.481	.143	72	28	.708	9.568	.647
780	.479	.142	72	35	.703	9.570	.649
790	.478	.141	72	42	.699	9.571	.651
800	.476	.140	72	49	.694	9.572	.653

$\varphi = 66°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.566	.227	66	11	.991	9.516	.564
110	.566	.227	66	15	.989	9.516	.564
120	.565	.227	66	19	.987	9.517	.565
130	.565	.226	66	23	.985	9.517	.566
140	.564	.226	66	27	.983	9.518	.567
150	.564	.225	66	31	.980	9.518	.568
160	.563	.224	66	35	.977	9.519	.569
170	.563	.223	66	39	.974	9.520	.570
180	.562	.222	66	43	.971	9.520	.571
190	.562	.221	66	47	.968	9.521	.572
200	.561	.220	66	51	.965	9.522	.573
210	.560	.219	66	55	.962	9.522	.575
220	.559	.218	67	00	.959	9.523	.576
230	.558	.217	67	05	.956	9.524	.577
240	.557	.216	67	10	.953	9.525	.578
250	.556	.215	67	15	.949	9.526	.580
260	.555	.214	67	20	.945	9.527	.581
270	.554	.213	67	25	.941	9.527	.582
280	.553	.211	67	31	.937	9.528	.584
290	.552	.210	67	37	.933	9.529	.585
300	.551	.209	67	43	.929	9.530	.586
310	.549	.207	67	49	.925	9.530	.588
320	.548	.206	67	55	.921	9.531	.589
330	.547	.204	68	01	.917	9.532	.590
340	.546	.203	68	07	.912	9.533	.592
350	.545	.201	68	13	.908	9.534	.593
360	.543	.200	68	20	.903	9.535	.595
370	.542	.199	68	27	.899	9.536	.596
380	.541	.197	68	34	.894	9.537	.598
390	.540	.195	68	41	.890	9.538	.599
400	.539	.194	68	48	.885	9.539	.601
410	.537	.192	68	55	.880	9.541	.603
420	.536	.191	69	02	.876	9.542	.604
430	.535	.189	69	09	.871	9.543	.606
440	.534	.188	69	16	.866	9.544	.608
450	.532	.186	69	23	.861	9.545	.610

$\varphi = 66°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.531	.185	69	31	.857	9.547	.612
470	.530	.183	69	38	.852	9.548	.613
480	.528	.182	69	45	.847	9.549	.615
490	.527	.180	69	53	.842	9.550	.617
500	.525	.179	70	01	.837	9.551	.619
510	.524	.177	70	08	.832	9.553	.621
520	.522	.176	70	16	.827	9.554	.623
530	.521	.174	70	23	.822	9.555	.625
540	.519	.173	70	30	.817	9.556	.627
550	.518	.171	70	37	.812	9.557	.629
560	.516	.170	70	44	.807	9.559	.632
570	.515	.168	70	51	.802	9.560	.634
580	.513	.167	70	58	.797	9.561	.636
590	.512	.165	71	05	.792	9.562	.638
600	.510	.164	71	12	.787	9.563	.640
610	.509	.162	71	19	.783	9.565	.643
620	.507	.161	71	26	.778	9.566	.644
630	.506	.159	71	33	.773	9.567	.646
640	.504	.158	71	40	.769	9.568	.649
650	.503	.157	71	47	.764	9.570	.651
660	.501	.155	71	54	.760	9.571	.653
670	.499	.154	72	00	.755	9.572	.655
680	.498	.153	72	07	.751	9.573	.657
690	.496	.151	72	14	.746	9.575	.659
700	.495	.150	72	21	.742	9.576	.661
710	.493	.148	72	28	.737	9.577	.664
720	.492	.147	72	35	.733	9.578	.566
730	.490	.145	72	42	.728	9.580	.668
740	.489	.144	72	49	.724	9.581	.670
750	.487	.143	72	56	.719	9.582	.672
760	.486	.141	73	03	.715	9.583	.674
770	.484	.140	73	10	.710	9.585	.677
780	.483	.138	73	17	.706	9.586	.679
790	.481	.137	73	23	.701	9.587	.681
800	.480	.136	73	30	.697	9.589	.683

$\varphi = 67°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{x}$	$\frac{y_0}{X}$
			°	′		—10	
100	.570	.219	67	10	.991	9.534	.592
110	.570	.219	67	14	.989	9.534	.592
120	.569	.219	67	18	.987	9.535	.593
130	.569	.218	67	21	.985	9.535	.594
140	.568	.218	67	25	.983	9.536	.595
150	.568	.217	67	29	.980	9.536	.596
160	.567	.216	67	33	.977	9.537	.597
170	.587	.215	67	37	.974	9.538	.598
180	.566	.214	67	41	.971	9.538	.599
190	.566	.213	67	45	.968	9.539	.600
200	.565	.213	67	49	.965	9.540	.601
210	.564	.212	67	53	.962	9.541	.603
220	.563	.211	67	57	.959	9.541	.604
230	.562	.210	68	01	.956	9.542	.605
240	.561	.209	68	06	.953	9.543	.606
250	.560	.208	68	11	.950	9.544	.608
260	.559	.207	68	16	.946	9.545	.609
270	.558	.205	68	21	.942	9.546	.610
280	.557	.204	68	26	.938	9.546	.612
290	.556	.203	68	32	.934	9.547	.613
300	.555	.202	68	38	.930	9.548	.615
310	.553	.200	68	44	.926	9.549	.616
320	.552	.199	68	50	.922	9.550	.618
330	.551	.198	68	56	.918	9.550	.619
340	.550	.196	69	02	.913	9.551	.621
350	.549	.195	69	08	.909	9.552	.622
360	.547	.194	69	14	.904	9.553	.624
370	.546	.192	69	20	.900	9.554	.625
380	.545	.191	69	26	.895	9.555	.627
390	.554	.190	69	33	.891	9.556	.628
400	.543	.188	69	40	.886	9.557	.630
410	.541	.187	69	47	.882	9.559	.632
420	.540	.185	69	54	.877	9.560	.634
430	.539	.184	70	01	.872	9.561	.636
440	.538	.182	70	08	.867	9.562	.638
450	.536	.181	70	15	.863	9.563	.639

$\varphi = 67°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.535	.179	70	22	.858	9.565	.641
470	.534	.177	70	29	.853	9.566	.643
480	.532	.176	70	36	.848	9.567	.645
490	.531	.174	70	44	.843	9.568	.647
500	.529	.173	70	51	.839	9.569	.649
510	.528	.171	70	58	.834	9.571	.651
520	.526	.170	71	05	.829	9.572	.653
530	.525	.168	71	12	.824	9.573	.656
540	.523	.167	71	19	.819	9.574	.658
550	.521	.165	71	26	.814	9.576	.660
560	.520	.164	71	32	.809	9.577	.662
570	.519	.162	71	39	.804	9.578	.664
580	.517	.161	71	46	.799	9.579	.666
590	.515	.160	71	52	.794	9.580	.668
600	.514	.159	71	59	.789	9.581	.671
610	.512	.157	72	05	.785	9.583	.673
620	.511	.156	72	12	.780	9.584	.675
630	.509	.154	72	19	.775	9.585	.677
640	.508	.153	72	25	.771	9.586	.679
650	.506	.152	72	32	.766	9.588	.681
660	.505	.150	72	39	.762	9.589	.684
670	.503	.149	72	45	.757	9.590	.686
680	.502	.147	72	52	.753	9.591	.688
690	.500	.146	72	58	.748	9.593	.690
700	.499	.145	73	05	.744	9.594	.693
710	.497	.143	73	12	.739	9.595	.695
720	.495	.142	73	18	.735	9.596	.697
730	.494	.141	73	25	.730	9.598	.699
740	.492	.139	73	31	.726	9.599	.701
750	.491	.138	73	38	.721	9.600	.704
760	.489	.137	73	44	.717	9.601	.706
770	.488	.135	73	51	.712	9.603	.708
780	.486	.134	73	58	.708	9.604	.711
790	.485	.133	74	04	.703	9.605	.713
800	.484	.132	74	11	.699	9.607	.715

$\varphi = 68°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.574	.211	68	10	.992	9.553	.622
110	.574	.211	68	13	.990	9.553	.623
120	.573	.211	68	17	.988	9.554	.624
130	.573	.210	68	20	.986	9.554	.625
140	.572	.210	68	24	.984	9.555	.626
150	.572	.209	68	28	.981	9.555	.627
160	.571	.209	68	31	.978	9.556	.628
170	.571	.208	68	35	.975	9.557	.629
180	.570	.207	68	39	.972	9.557	.630
190	.570	.206	68	43	.969	9.558	.631
200	.569	.205	68	47	.966	9.559	.632
210	.568	.204	68	51	.963	9.560	.634
220	.567	.203	68	55	.960	9.560	.635
230	.566	.202	68	59	.956	9.561	.636
240	.565	.201	69	03	.954	9.562	.638
250	.564	.200	69	08	.951	9.563	.639
260	.563	.199	69	13	.947	9.564	.640
270	.562	.198	69	18	.943	9.565	.642
280	.561	.197	69	23	.939	9.565	.643
290	.560	.196	69	28	.935	9.566	.645
300	.559	.195	69	33	.931	9.567	.646
310	.557	.193	69	39	.927	9.568	.648
320	.556	.192	69	45	.923	9.569	.649
330	.555	.191	69	51	.919	9.569	.651
340	.554	.190	69	57	.914	9.570	.652
350	.553	.188	70	03	.910	9.571	.654
360	.551	.187	70	09	.905	9.572	.655
370	.550	.186	70	15	.901	9.573	.657
380	.549	.185	70	21	.896	9.574	.658
390	.548	.183	70	27	.892	9.575	.660
400	.547	.182	70	33	.887	9.576	.662
410	.545	.181	70	39	.883	9.577	.664
420	.544	.179	70	45	.878	9.578	.666
430	.543	.178	70	52	.873	9.579	.668
440	.542	.176	70	59	.869	9.580	.670
450	.540	.175	71	06	.864	9.581	.672

$\varphi = 68°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.539	.173	71	13	.859	9.583	.674
470	.538	.172	71	20	.855	9.584	.676
480	.536	.170	71	27	.850	9.585	.678
490	.535	.169	71	34	.845	9.586	.680
500	.533	.167	71	41	.841	9.587	.682
510	.532	.166	71	48	.836	9.589	.685
520	.530	.165	71	55	.831	9.590	.687
530	.529	.163	72	01	.826	9.591	.689
540	.527	.162	72	08	.821	9.592	.691
550	.526	.161	72	14	.816	9.593	.693
560	.524	.159	72	20	.811	9.595	.696
570	.523	.158	72	27	.806	9.596	.698
580	.521	.157	72	33	.801	9.597	.700
590	.520	.155	72	39	.796	9.598	.702
600	.518	.154	72	46	.791	9.599	.704
610	.517	.153	72	52	.787	9.601	.707
620	.515	.151	72	58	.782	9.602	.709
630	.514	.150	73	05	.777	9.603	.711
640	.512	.148	73	11	.773	9.604	.713
650	.511	.147	73	17	.768	9.606	.715
660	.509	.146	73	24	.764	9.607	.718
670	.508	.144	73	30	.759	9.608	.720
680	.506	.143	73	36	.755	9.609	.722
690	.505	.142	73	42	.750	9.611	.724
700	.503	.140	73	49	.746	9.612	.726
710	.502	.139	73	55	.741	9.613	.729
720	.500	.138	74	01	.737	9.614	.731
730	.499	.137	74	07	.732	9.616	.733
740	.497	.135	74	14	.728	9.617	.735
750	.496	.134	74	20	.723	9.618	.738
760	.494	.133	74	26	.719	9.619	.740
770	.492	.132	74	33	.714	9.621	.742
780	.491	.130	74	39	.710	9.622	.745
790	.489	.129	74	45	.705	9.623	.747
800	.487	.128	74	52	.701	9.625	.750

$\varphi = 69°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.578	.203	69	10	.992	9.572	.655
110	.578	.203	69	13	.990	9.572	.655
120	.578	.203	69	16	.988	9.573	.656
130	.577	.202	69	20	.986	9.573	.657
140	.577	.202	69	23	.984	9.574	.658
150	.576	.201	69	26	.981	9.575	.659
160	.576	.201	69	30	.978	9.576	.660
170	.575	.200	69	33	.975	9.577	.661
180	.575	.200	69	36	.972	9.578	.663
190	.574	.199	69	40	.969	9.578	.664
200	.573	.198	69	44	.966	9.579	.665
210	.572	.197	69	48	.963	9.580	.666
220	.571	.196	69	52	.960	9.580	.667
230	.570	.195	69	56	.957	9.581	.669
240	.569	.194	70	00	.954	9.582	.670
250	.568	.193	70	04	.951	9.583	.671
260	.567	.192	70	09	.948	9.583	.673
270	.566	.191	70	14	.944	9.584	.674
280	.565	.190	70	19	.940	9.585	.676
290	.564	.189	70	24	.936	9.586	.677
300	.563	.188	70	29	.932	9.587	.679
310	.561	.186	70	34	.927	9.588	.681
320	.560	.185	70	39	.923	9.589	.682
330	.559	.184	70	45	.919	9.590	.684
340	.558	.183	70	51	.915	9.590	.686
350	.557	.182	70	57	.910	9.591	.687
360	.556	.180	71	03	.906	9.592	.689
370	.554	.179	71	09	.901	9.593	.691
380	.553	.178	71	15	.897	9.594	.692
390	.552	.177	71	21	.892	9.594	.694
400	.551	.175	71	27	.888	9.595	.696
410	.549	.174	71	33	.883	9.596	.698
420	.548	.173	71	39	.879	9.597	.700
430	.547	.172	71	45	.874	9.598	.702
440	.545	.170	71	51	.870	9.599	.704
450	.544	.169	71	57	.865	9.600	.706

$\varphi = 69°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		−10	
460	.543	.167	72	04	.861	9.601	.708
470	.541	.166	72	10	.856	9.602	.710
480	.540	.164	72	16	.852	9.603	.712
490	.538	.163	72	23	.847	9.605	.715
500	.537	.161	72	29	.843	9.606	.717
510	.535	.160	72	35	.838	9.607	.719
520	.533	.159	72	42	.833	9.608	.721
530	.532	.158	72	48	.828	9.609	.724
540	.530	.156	72	54	.823	9.610	.726
550	.529	.155	73	00	.818	9.612	.728
560	.527	.154	73	07	.813	9.613	.730
570	.526	.152	73	13	.808	9.614	.733
580	.524	.151	73	19	.803	9.615	.735
590	.523	.150	73	25	.798	9.616	.737
600	.521	.148	73	37	.793	9.618	.740
610	.520	.147	73	38	.789	9.619	.742
620	.518	.146	73	44	.784	9.620	.744
630	.516	.144	73	50	.779	9.621	.746
640	.515	.143	73	56	.775	9.623	.749
650	.513	.142	74	02	.770	9.624	.751
660	.512	.141	74	09	.765	9.625	.753
670	.510	.139	74	15	.761	9.626	.756
680	.509	.138	74	21	.756	9.627	.758
690	.507	.137	74	27	.752	9.628	.760
700	.506	.135	74	33	.748	9.630	.763
710	.504	.134	74	39	.743	9.631	.765
720	.502	.133	74	45	.739	9.632	.768
730	.500	.132	74	51	.734	9.634	.770
740	.499	.130	74	57	.730	9.635	.773
750	.498	.129	75	03	.725	9.636	.775
760	.496	.128	75	08	.721	9.637	.778
770	.495	.127	75	14	.716	9.639	.780
780	.493	.125	75	20	.712	9.640	.783
790	.492	.124	75	26	.707	9.642	.785
800	.490	.123	95	32	.703	9.644	.788

$\varphi = 70°$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{v_\omega}{V}$	$\log\frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
100	.582	.194	70	10	.993	9.592	.690
110	.582	.194	70	13	.991	9.592	.690
120	.582	.194	70	16	.989	9.582	.691
130	.581	.193	70	19	.988	9.593	.692
140	.581	.193	70	22	.985	9.593	.693
150	.580	.192	70	25	.982	9.594	.694
160	.580	.192	70	28	.979	9.594	.695
170	.579	.191	70	31	.976	9.595	.696
180	.579	.191	70	34	.973	9.595	.698
190	.578	.190	70	38	.970	9.596	.699
200	.577	.189	70	42	.967	9.597	.700
210	.577	.188	70	46	.964	9.597	.702
220	.576	.188	70	50	.961	9.598	.703
230	.575	.187	70	54	.958	9.598	.705
240	.574	.186	70	58	.955	9.599	.706
250	.573	.185	71	02	.952	9.600	.708
260	.572	.184	71	06	.949	9.601	.709
270	.570	.184	71	10	.945	9.602	.711
280	.569	.183	71	15	.941	9.603	.712
290	.568	.182	71	20	.937	9.604	.714
300	.567	.181	71	25	.933	9.605	.715
310	.566	.180	71	30	.928	9.605	.717
320	.565	.179	71	35	.924	9.606	.719
330	.563	.178	71	40	.920	9.607	.720
340	.562	.177	71	45	.916	9.608	.722
350	.561	.175	71	51	.911	9.609	.724
360	.560	.174	71	56	.907	9.610	.726
370	.558	.173	72	02	.902	9.611	.727
380	.557	.172	72	08	.898	9.612	.729
390	.556	.170	72	14	.893	9.613	.731
400	.554	.169	72	19	.889	9.614	.733
410	.553	.168	72	25	.884	9.615	.735
420	.552	.166	72	31	.880	9.616	.737
430	.550	.164	72	37	.875	9.617	.740
440	.549	.163	72	43	.871	9.618	.742
450	.548	.162	72	49	.866	9.619	.744

$\varphi = 70^\circ$

$\frac{V}{\sqrt{C}}$	$\frac{10T}{V}$	$\frac{10X}{V^2}$	ω		$\frac{V_\omega}{V}$	$\log \frac{h}{X}$	$\frac{y_0}{X}$
			°	′		—10	
460	.546	.161	72	55	.862	9.620	.746
470	.545	.160	73	00	.857	9.622	.749
480	.543	.158	73	06	.853	9.623	.751
490	.542	.157	73	12	.848	9.624	.753
500	.540	.155	73	18	.844	9.625	.755
510	.539	.154	73	24	.840	9.626	.758
520	.537	.152	73	30	.835	9.627	.760
530	.536	.151	73	36	.830	9.629	.762
540	.534	.150	73	42	.825	9.630	.765
550	.532	.148	73	48	.820	9.631	.767
560	.530	.147	73	54	.815	9.632	.769
570	.529	.146	74	00	.810	9.633	.772
580	.527	.144	74	06	.805	9.635	.774
590	.526	.143	74	12	.800	9.636	.777
600	.524	.142	74	18	.795	9.637	.779
610	.523	.141	74	24	.791	9.639	.781
620	.522	.139	74	30	.786	9.640	.784
630	.520	.138	74	36	.781	9.642	.786
640	.519	.137	74	42	.777	9.643	.789
650	.517	.136	74	48	.772	9.644	.791
660	.515	.135	74	54	.767	9.645	.794
670	.514	.133	75	00	.763	9.646	.796
680	.512	.132	75	06	.758	9.648	.799
690	.511	.131	75	12	.754	9.649	.801
700	.509	.130	75	18	.750	9.650	.803
710	.508	.129	75	23	.745	9.651	.806
720	.506	.128	75	29	.741	9.653	.808
730	.505	.126	75	35	.736	9.654	.811
740	.503	.125	75	40	.732	9.655	.813
750	.501	.124	75	45	.727	9.656	.816
760	.508	.123	75	50	.723	9.658	.818
770	.498	.122	75	56	.718	9.659	.821
780	.496	.121	76	01	.714	9.660	.823
790	.495	.119	76	07	.709	9.662	.826
800	.493	.118	76	12	.705	9.663	.828

APPENDIX IX.

THE DRIFT OF RIFLED PROJECTILES*

The drift for direct fire guns is comparatively small, and rough approximations frequently suffice for the ordinary purposes of gunnery. When, however, the methods used in direct fire are applied to high angle fire, the lack of generality of empirical or approximate formulas becomes evident.

The angular drift in high angle fire may become of very considerable magnitude, and, moreover, at very great elevations its behavior is such as to cause astonishment, unless the nature of its double-curved trajectory is comprehended. The drift beyond certain elevations (for example, 66° in the steel mortar) may diminish more or less rapidly and reach zero, crossing the plane of fire.

The purpose of this paper is to furnish a drift formula of simple form independent of any explicit hypothesis as to the law of retardation. or as to the character of fire (whether direct, curved or high angle), the formula to be generally applicable to practical problems of drift.

The character of the double-curved path of the projectile in the air, giving rise to the peculiar phenomena noted at very high angles of departure deserves discussion. (See "Report of the Chief of Ordnance, 1904," for observed phenomena of this kind.)

It is not, however, within the compass of this paper to enter into the details of the analytical treatment of this subject; nor is it fitting, since the writer could not do better than quote the admirable work of Major P. Charbonnier, of the French Artillery. ("Problèmes balistiques secondaires," 1907; pp. 281 to 389 inclusive.) The discussion will, therefore, be limited to the facts of the case, leaving the reader to study Major Charbonnier's work, if the details are desired.

The forms of the working equations for drift given in Charbonnier's work are different for each kind of fire and involve hypotheses regarding each. In the present paper the writer will deduce a formula applicable to all kinds of fire and in simple form. It is believed to be the first of its kind.

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* See Journal of the U. S. Artillery, July-August, 1907.

When a mortar is given a quadrant elevation exceeding a value approximately fixed for a given mortar and projectile (though somewhat affected by the muzzle velocity), the point of fall of the projectile becomes uncertain, from a practical point of view, so far as deviation or deflection is concerned, though *the range is not sensibly affected*. This effectually disposes of the theory that the projectile presents its base to the air resistance, if such proof is necessary. Observers, indeed, say that its point is always foremost ; and it is hard to conceive of its maintaining its base to the front for any length of time under the conditions. The fact frequently observed that its point is slightly to the right or left of its base as it descends, is entirely in accord with the theory of precession of the extremity of the projectile's axis, and is a natural phenomenon to be expected. Thus, so far as range is concerned, we might continue to elevate beyond this critical limit ; but the uncertainty as to where the projectile would fall in azimuth effectually puts a superior limit to practicable elevations for the particular system.

The following facts regarding the behavior of a projectile in flight, will, it is believed, prove interesting :—

1. The projectile does *not* behave exactly as does a top or gyroscope, because the projectile suffers a great and variable resistance in its path.

2. The resultant resistance offered to a projectile in its flight, is, while nearly in the direction of its axis, sufficiently inclined to it and to the path of the center of gravity of the projectile, to make these facts of great importance in the determination of the drift.

3. The problem is simplified by the fact that modern projectiles are solids of revolution, if carefully constructed, in which case the ellipsoid of inertia is a prolate spheroid.

4. The results of the forces acting are distinguished as follows :—

   (a) A retarding force ;
   (b) A deviating force ;
   (c) A disturbing couple.

5. The shape of the head is of importance in this connection. K varies approximately as the square of the coefficient of form or as $\frac{4n-1}{n^2}$ in which n is the radius of the ogive in calibers. The position of the center of gravity relatively to the center of resistance is of great importance in the discussion of the drift.

6. The radius of gyration of the projectile around its longitudinal axis also plays an important part.

7. In the deduction of any expression giving the drift, it is essential to consider the angle made by the axis of the projectile with the tangent to the trajectory as compared with the angle made by the resultant resistance with the axis of the projectile. These angles are both small by hypothesis, since, if they are not so, it is at once evident that the flight of the projectile is so unsteady as to render it unfit for use. Though the ratio, denoted by K, of the former to the latter is not fixed nor determinable *a priori* with certainty, its average values *for our service projectiles* may be taken, in the absence of experimental firings, to be as follows :—

Steel D.P. shell	K = 0.75
Solid shot C. I. or steel	
C.I. shell	K = 0.80 or more.

The difference in the case of C. I. shell is probably due to imperfect centering and finish. If the momental ellipsoids were the same, the C. I. and steel shells should behave alike. These values assume sufficient steadiness of flight to meet the requirements of the hypothesis made above.

8. The lateral trajectory of a projectile in its flight through the air is a roulette, which is shown by Charbonnier to have the following characteristics :—

(a) Where the velocity is infinite, the radius of curvature vanishes.

(b) The radius of curvature continually increases in the ascending branch.

(c) The radius of curvature reaches a maximum value between the summit and the point of least velocity.

9. In direct fire, the point of minimum curvature of the roulette is beyond the point of fall, but in high angle fire with high elevations it is, in general, above the point of fall and between the latter and the summit. Hence, in high angle fire alone, of the two, would we expect a maximum drift, decreasing thence more or less sharply as the elevation increases.

10. In considering the drift for practical purposes, we are not concerned with the wave-like character of the curve where the amplitude is small, but we take a curve giving their mean, and forming as it were, the bent axis of a helix of varying radius and pitch. This restriction eliminates the consideration of the drift after the amplitude of the convolutions becomes relatively large ; but as this latter takes place only when the upper limit

of elevation is reached, it is of no practical moment, and the drift is capable of treatment by such a method within the limits of practice. It is also highly likely that the value of K is altered materially at elevations beyond the limiting ones, especially with low muzzle velocities.

11. That portion of the curve of precession which lies beyond the point of minimum curvature, is such that the projectile will behave as to drift in what may, for practical purposes, be called a very erratic way. Analytically, it may be explained, but it is evident that a slight change in elevation may produce a great change in the drift. Under certain conditions, the character of the curve may be such that this erratic behavior is deferred beyond 70°, but the return to the plane of fire will have begun. The 75 mm. subcaliber tube gives an illustration of this.

To deduce a working formula, the expression for the acceleration across the plane of fire due to the deviating force is taken to be

$$\frac{dv_z}{dt} = \left\{ 1 - K \right\} \cdot \frac{2gV}{n} \cdot \frac{d^3}{w} \cdot \frac{\cos\theta}{v} . \qquad (1)$$

in which $\frac{d^3}{w}$ is found to absorb in itself the general constants ordinarily involved, the expression involving the radius of gyration in calibers, and the distance between the center of pressure and the center of gravity, (also in calibers). This is borne out by the factor $\frac{d}{C}$ in Hélie's empirical formula, which was found without the aid of analysis. V is the muzzle velocity in feet per second ; n the reciprocal of the final twist (1 turn in n calibers) ; d the caliber in inches ; w the weight of the projectile in pounds ; θ the inclination to the horizontal of the tangent to the trajectory at the point considered, and v the remaining velocity in feet per second.

The acceleration along the radius of curvature in the plane of fire is given by

$$\frac{v d\theta}{dt} = - g \cos\theta \qquad (2)$$

in which the law of retardation does not explicitly appear.

Combining (1) and (2), and eliminating $\frac{g\cos\theta}{v}$, we find

$$\frac{dv_z}{dt} = -(1-K) \cdot \frac{2V}{n} \cdot \frac{d^3}{w} \cdot \frac{d\theta}{dt}$$

from which

$$v_z = (1 - K) \cdot \frac{2V}{n} \cdot \frac{d^3}{w} \cdot (\varphi - \theta). \qquad (3)$$

This is the lateral velocity at the point considered.

Taking the mean value between φ and θ, and designating the lateral distance gained by z, we find (t being the time of flight to θ),

$$\frac{z}{t} = (1 - K) \frac{Vd^3}{wn} \cdot (\varphi - \theta).$$

From which, multiplying through by $\frac{t}{x}$, we have

$$\frac{z}{x} = (1 - K) \frac{d^3}{wn} \cdot \frac{(\varphi - \theta)\,Vt}{x} \qquad (4)$$

Thus, if left to the deviating force alone, the projectile would have moved in deflection an angle whose radian measure is $\frac{z}{x}$. But in the act of obeying the deviating force, the projectile encounters a resistance producing a retardation, which is the lateral component of the total horizontal retardation; and, as the resulting deflection, due to the two causes, is the drift, (which we will designate D), we find that if r_x is the mean horizontal retardation, r_x sin D is the mean lateral retardation. Now, since D is small and in radians it may be substituted for its sine, and the cross retardation then becomes Dr_x. Since the time of flight from φ to θ is the same for both retardations, the distance effects should be proportional to the mean retardations. Hence they are in the ratio D: 1. But the loss of range due to r_x is Vt cos φ — x, and hence that due to r is D(Vt cos φ — x). Reducing this to radians, the pole being at the origin, we have as its value

$$D \left\{ \frac{Vt \cos \varphi}{x} - 1 \right\}$$

Hence, finally

$$D = \frac{z}{x} - D \left\{ \frac{Vt \cos \varphi}{x} - 1 \right\}.$$

or,

$$D = \frac{z}{x} \cdot \frac{x}{Vt \cos \varphi}.$$

Substituting for $\frac{z}{x}$ its value from (4), we find

$$D = (1 - K) \frac{d^3}{wn} \cdot \frac{(\varphi - \theta)Vt}{x} \cdot \frac{x}{Vt \cos \varphi},$$

or,

$$D = (1 - K) \frac{d^3}{wn} \cdot (\varphi - \theta) \sec \varphi \qquad (5)$$

which, for the point of fall, becomes

$$D = (1 - K) \frac{d^3}{wn} \cdot (\varphi + \omega) \sec \varphi \qquad (6)$$

and since D and $\varphi + \omega$ are *both* in radians, it is plain that we may take them in degrees or minutes, provided only that *both* are so taken.

An inspection of the formula just deduced will show how much ground there is for the statement so frequently heard (notably among our experienced field officers), and recognized at the Proving Ground at Sandy Hook as an empirical fact, that "the angular drift is dependent on the elevation alone" for a given mortar and projectile.

An examination of data shows that in high angle fire this is very nearly true for a given projectile, whatever the velocity, so long as steadiness is secured; and we see from formula (6) that, since φ and ω in high angle fire are not far different, the drift depends on φ alone, for practical purposes.

In direct fire, however, ω / φ is much greater than unity and they cannot be taken as equal, and ω must be found.

Formula 6 (if φ and ω be assumed equal, and any error in this assumption be absorbed into K) will become

$$D = 2(1 - K) \frac{d^3}{wn} \cdot \varphi \sec \varphi \qquad (6\ m)$$

and a mean value of K may be found using the data for light winds and long ranges by preference, this form of the formula being restricted to high angle fire, and giving a mean drift for each elevation, independent of the zone.

In this connection the following table will be useful :—

VALUES OF φ SEC φ

φ	φ sec φ	log (φ sec φ)	φ	φ sec φ	log (φ sec φ)
°	°		°	°	
45	63.64	1.8037	56	100.15	2.0006
46	66.22	1.8210	57	104.66	2.0198
47	68.92	1.8383	58	109.45	2.0392
48	71.74	1.8557	59	114.55	2.0590
49	74.69	1.8733	60	120.00	2.0792
50	77.79	1.8910	61	125.82	2.0998
51	81.04	1.9087	62	132.06	2.1208
52	84.46	1.9267	63	138.77	2.1423
53	88.07	1.9448	64	146.00	2.1643
54	91.87	1.9632	65	153.80	2.1870
55	95.89	1.9818			

A few examples of the use of this formula will be given :—

EXAMPLE 1:—12-inch C. I. Mortar, steel hooped, $w = 1046$, D. P. steel shell. Find drift *a priori* (assumingK $= 0.75$) for the following elevations : 45°, 50°, 55°, 59°, 62°, and compare the results with those actually observed.* ($n = 25$)

RESULTS

Calculated :	2° 6′	2° 34′	3° 10′	3° 47′	4° 11′
Observed :	2° 10′	2° 28′	2° 57′	3° 48′	4° 43′
Ratios :	1.03	0.96	0.93	1.00	1.08
Average ratio :	1.00				

Hence $K = 0.75$ as assumed.

EXAMPLE 2 :—D. P. steel shell, $n = 20$, $w = 1028$ lbs., $V = 1150$, find drift for $\varphi = 65^\circ$. (*a priori*, taking $K = 0.75$, as before)—and compare with observed data.

Result :—Drift ° = 6° 28′.

Observed drift 5° 59′, with a 7-mile wind blowing from right rear. Hence drift would appear to be greater than 5° 59′. 6° 28′ is near the truth.

EXAMPLE 3 :—Steel mortar, C. I. shell (uncapped), $w = 1000$ lbs., $n = 20$, $\varphi = 45^\circ$, $V = 1000$ f.s., ($K = 0.80$).

Drift (*a priori*) 2° 12′.

Observed value 2° 15′.

EXAMPLE 4 :—12-inch B. L. Rifle, Model 1900, $w = 1046$, $n = 25$, $\phi = 6^\circ\ 26'$ $\omega = 8^\circ\ 54'$. Find angular drift for solid shot, C. I., and compare with observed drift ($K = 0.75$).

Result :—Drift $= 15'.33$.

Average for the 5 shots of the group when corrected for wind, 16′.

EXAMPLE 5 :—The mortar subcaliber tube (75 mm.) gives by plotting from actual firings for all zones the following average results for the 18-lb. projectile ($n = 29.83$), (cast iron).

φ	
45°	1°.14
55°	1°.71
65°	2°.75

(a) See if K is constant.

(b) Find K.

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* The wind in the upper strata was not known and there were light surface winds. The calculated drifts neither exceed nor fall short of all or a majority of the observed values and the *average* is the same. The two shots fired with elevations 64° and 65° in this group and not quoted give unmistakable evidence of a *negative jump* of several degrees, from an inspection of the ranges, times of flight and drift—which correspond. The same is noted with the C I. projectile at this velocity (950 f.s.) with regard to these two elevations. The superior limit of elevation should therefore for this velocity be fixed at 62°. (See "Report of Chief of Ordnance, 1905," for verification of the *observed* facts.)

(a) Assuming K constant we should have the drift proportional to $\phi \sec \phi$. Hence we should find

$$\frac{63.64}{1.14} = \frac{95.89}{1.71} = \frac{153.80}{2.75}$$

Now each of these is sensibly equal to 56.0. Hence K is constant.

(b) $2.75 = (1 - K)\frac{2d^3}{wn} \cdot 65° \sec 65°,$

or $1 - K = .186$

Hence $K = .814$

This last example shows best the use of the formula in adjusting observed values of the drift. K may not always prove absolutely constant and this may be due to one or more of several causes: faulty observations; a slight wave in the curve; unknown upper winds. The first is common to direct and high angle fire. The second manifests itself mainly in practice in direct fire; the third is peculiar to high angle fire. Hence exact accord is impracticable and several observations at different elevations should be used as a basis in finding the value of K. A good rule is to select observed drifts with little wind, (for which correct,) and take the average of the K's found, using the following method :—

Multiply each (1 — K) by its range and add the products. Divide by the sum of the ranges. The resulting mean value of K is then

$$K = 1 - \frac{\Sigma(1 - K)R}{\Sigma R}$$

This gives greater weight to the values of K at the longer ranges where the angular drift produces lateral deviations in proportion to the range, and thus secures the best mean.

Whenever the wind is known throughout the trajectory with good approximation, allowance should be made for its effect before summarizing the drift results by deducing K. In order to do this the formula

$$\text{Deflection (degrees)} = \frac{84W}{V \cos \varphi}\left\{\frac{V\,T \cos \varphi}{X} - 1\right\}$$

should be used. In this formula W is the transverse wind component in miles per hour; X, the range in feet.

The determination of K for observed data with regard to the 3-inch field gun model of 1905, will be a practical example of the use of this method. In the data given below, the deviation is in yards and is negative if to the left of the plane of fire. A cross wind from right to left is affected with a negative sign.

The mean data of the four groups are given as follows :—

	Range yds.	Deviation yds.	Time of flight sec.	Cross-wind mi. per hr.	ϕ	ω
a	3126	— 5.63	8.1	—14.5	5°.33	7°.83
b	3964	+21.35	11.2	— 1.4	7°.99	12°.27
c	5005	+39.14	14.8	2.5	11°.22	17°.52
d	5940	+49.46	19.0	— 4.1	15°.03	23°.12

Calculated values are tabulated as follows : –

	Wind deflection degrees effect	Wind deflection degrees correction	Mesured deflection degrees	Drift for no wind degrees	$\phi + \omega$
a	—0°.32	+0°.32	—0°.10	0°.22	13°.16
b	—0°.04	+0°.04	+0°.31	0°.35	22°.26
c	—0°.09	+0°.09	+0°.45	0°.54	28°.74
d	—0°.16	+0°.16	+0°.48	0°.64	38°.15

From the formula for drift

$$\text{Drift}^\circ = (1 - K)\frac{d^3}{wn}(\phi + \omega)\sec\phi = .072\,(1 - K)(\phi + \omega)\sec\phi$$

Hence $$1 - K = \frac{\text{Drift}^\circ}{.072\,(\phi + \omega)\sec\phi}$$

Substituting the corrected angular drift from the fourth column of the table last given, we find as values of K for the different groups

(a) .7688; (b) .7623; (c) 7440; (d) .7749

Taking the ranges to the nearest hundred yards we have for $(1 - K)\,R$

(a) .2312 × 3100 717
(b) .2377 × 4000 951
(c) .2560 × 5000 1280
(d) .2251 × 5900 1328

$\Sigma R = 18000$; $\Sigma(1 - K)\,R = 4276$

Hence $$1 - K = \frac{\Sigma(1 - K)\,R}{\Sigma R} = \frac{4276}{18000} = .2375$$

or $K = .7625$

Using this value we find the equation of drift for this gun and projectile

$$\text{Drift} = .071\,(\phi + \omega)\sec\phi = [8.2330 - 10]\,(\phi + \omega)\sec\phi.$$

This value of K is not unexpected, since a shrapnel is practically a solid shot except for the fact that it has the attributes of a shell near its base where the charge is; and the interior metal (bullets and separators), while centered, does not behave exactly as if the projectile were of one solid piece of metal. Hence we should expect to find K near 0.75, its value for solid shot.

This example furnishes a model for the treatment of data for direct fire.

For high angle fire the wind formula may be used with discrimination, as we have no assurance that the wind is constant in direction and velocity. Data should be selected with little or no wind and graphic methods are of value in adjusting data before determining the drift constant.

The examples seem sufficiently varied, embracing cast iron and steel projectiles, shot, shell and shrapnel ; twists 1 in 20, 1 in 25 and 1 in 29.83 ; capped and uncapped projectiles, direct and high angle fire, very large and very small projectiles. The observed values quoted are from firings at Sandy Hook Proving Ground ; and were either averages of all shots fired, or selected on account of absence of heavy winds. The formula deduced places us now in a position not only to say that the angular drift for a gun system depends on φ and ω alone in direct fire, and on φ alone in high angle fire, but gives an expression for its value in either case, thus determining the shape of a drift curve in the surest manner and supplying missing values in case of high winds or of failure in observation ; and even in many cases furnishing an *a priori* solution.

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